Lesson 44 Review 2

Final Exam: Monday, Dec. 14, 7-9 pm Loeb Playhouse (STEW 183)

3 crib sheets handwritten both sides (keep)

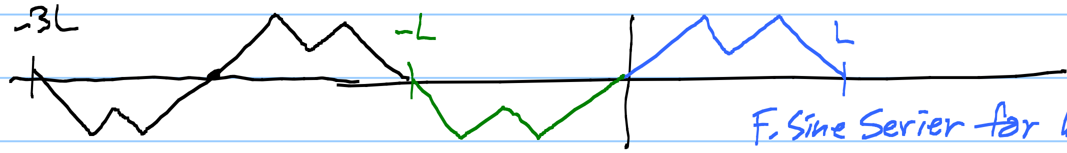
Laplace Transform Table on <u>last page</u>

Off. Hrs. F, M 2-3pm

WebEX Sunday 8-9 pm

Fourier Series $[-L, L]$

F. Sine, Cosine Series $[0, L]$



F. Sine Series for blue is the <u>Full</u> F. Series for <u>odd</u> periodic ext.

18., 19. $\sum$ Cosine = even

$\sum$ Sine = odd

S-L prob. Find all <u>positive</u> e-vals for S-L prob:

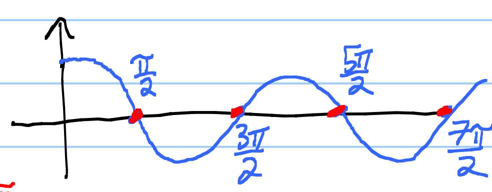
$$\begin{cases} X'' + \lambda X = 0 \\ X(0) = 0,\ X'(L) = 0 \end{cases}$$

$\lambda = \mu^2 \quad X = \cancel{c_1 \cos \mu x} + \underline{\underline{c_2 \sin \mu x}}$

$X(0) = c_1 \overset{\text{want}}{=} 0$

$X'(L) = c_2 \mu \cos \underbrace{\mu L}_{= (\text{odd})\frac{\pi}{2}} = 0$

$\cos\theta$



$\frac{\pi}{2} + n\pi$

$\mu L = n\frac{\pi}{2} \quad n = 1, 3, 5, 7, \ldots$

$\lambda = \mu^2 = \left(\frac{n\pi}{2L}\right)^2 \quad n = 1, 3, 5, \ldots \qquad c_1 = 0$, Take $c_2 = 1$.

$X_n = \sin \frac{(2n-1)\pi}{2L} x \quad n = 1, 2, 3, \ldots \leftarrow \perp$ on $[0, L]$

$f(x) = \sum_{n=1}^{\infty} c_n \sin \frac{(2n-1)\pi}{2L} x \leftarrow$ mult by $\sin \frac{(2m-1)\pi}{2L} \cdot \int_0^L$

Heat prob: iced 0 ——— L insulated

$$u(x,t) = \sum_{n=1}^{\infty} c_n \sin \frac{(2n-1)\pi}{2L} x\, e^{-c^2 \left(\frac{(2n-1)\pi}{2L}\right)^2 t}$$

Crib sheet: Heat, Wave on $[0, L]$ with <u>Homog</u> BC

Fourier Integral:

$$f(x)=\int_0^\infty A(w)\cos wx + B(w)\sin wx \, dw$$

where
$$\begin{cases} A(w)=\frac{1}{\pi}\int_{-\infty}^{\infty} f(p)\cos wp \, dp \\ B(w)=\frac{1}{\pi}\int_{-\infty}^{\infty} f(p)\sin wp \, dp \end{cases}$$

Heat prob on $\mathbb{R}$:
$$\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}$$

$$u(x,0)=f(x) \quad \text{IC}$$

$$u(x,t)=\int_0^\infty \left(A(w)\cos wx + B(w)\sin wx\right)e^{-c^2w^2t}\,dw$$

$u(x,0)$ = F.I. for $f(x)$! Prob. 27

A, B come from F.I. for f.

Crib sheet: Fourier Sine, Cosine Trans., Complex F.T.

$\tilde{\mathcal{F}}_s[f]$ $\qquad \tilde{\mathcal{F}}_c[f]$ $\qquad \hat{f}$

$\hat{f}_s$ $\qquad \hat{f}_c$ $\qquad \tilde{\mathcal{F}}[f]$

Complex Fourier Series not on exam, but $\hat{f}$ maybe.

Rules: $\tilde{\mathcal{F}}_c[f']$, $\tilde{\mathcal{F}}_c[f'']$, $\tilde{\mathcal{F}}_s$ ..., $\tilde{\mathcal{F}}$

Wave eqn: String
$$\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2} \qquad \begin{cases} u(0,t)=0 \\ u(L,t)=0 \end{cases} \text{BC}$$

$$\begin{cases} u(x,0)=f(x) \\ \frac{\partial u}{\partial t}(x,0)=g(x) \end{cases} \text{IC}$$

1) Sep. of vars. Fourier Series sol^n

$$u(x,t)=\sum_{n=1}^\infty \sin\frac{n\pi x}{L}\left(A_n\cos\frac{cn\pi t}{L}+B_n\sin\frac{cn\pi t}{L}\right)$$

A_n Fourier Sine Series coeff for f

$\frac{cn\pi}{L} B_n$ Fourier Sine Series coeff for g.

2) D'Alembert's sol^n. Take odd periodic ext of f and g.

$$u(x,t) = \varphi(x+ct) + \psi(x-ct)$$
$$= \frac{1}{2}\left[f(x+ct) + f(x-ct)\right]$$
$$+ \frac{1}{2c}\left[G(x+ct) - G(x-ct)\right]$$

where $G(x) = \int_0^x g(u)\,du$.

$$\boxed{u(x,t) = \frac{1}{2}\left[f(x+ct)+f(x-ct)\right] + \frac{1}{2c}\int_{x-ct}^{x+ct} g(u)\,du}$$

works for ∞ string!

26. $u_{tt} = \underset{c=2}{4}\, u_{xx}$ on $\mathbb{R}$ $\quad u(x,0) = \sin x$

$\frac{\partial u}{\partial t}(x,0) = \cos x$

$$u(x,t) = \varphi(x+ct) + \psi(x-ct) \qquad u(x,0) = \varphi(x) + \psi(x) \overset{\text{want}}{=} \sin x \qquad (A)$$

$$\frac{\partial u}{\partial t}(x,t) = c\,\varphi'(x+ct) - c\,\psi'(x-ct) \qquad \frac{\partial u}{\partial t}(x,0) = 2\varphi'(x) - 2\psi'(x) \overset{\text{want}}{=} \cos x \qquad (B)$$

$$\to \varphi'(x) - \psi'(x) = \frac{1}{2}\cos x$$

anti diff

$$\varphi(x) - \psi(x) = \frac{1}{2}\sin x \qquad (B')$$

$(A) + (B')$: $\quad \varphi(x) = \frac{3}{4}\sin x$

$(A) - (B')$: $\quad \psi(x) = \frac{1}{4}\sin x$

$$\underline{u(x,t) = \frac{3}{4}\sin(x+2t) + \frac{1}{4}\sin(x-2t)}$$

Keep straight: Convolution for $\mathcal{L}$ ← know

for $\tilde{\pi}$ ← not on exam

Dirichlet Problem on disc: Solve $\underbrace{\Delta u = 0}_{u \text{ harmonic}}$

BC $\quad u(R,\theta) = f(\theta)$ ← given

$$u(r,\theta) = a_0 + \sum_{n=1}^{\infty}\left(A_n \left(\frac{r}{R}\right)^n \cos n\theta + B_n \left(\frac{r}{R}\right)^n \sin n\theta\right)$$

$u(R,\theta) =$ Fourier Series for $f(\theta)$.

EX: $\quad f(\theta) = 3 + 5\cos 2\theta + 7\sin 13\theta$ ← Is a F. Series!

$$u(r,\theta) = 3 + 5\left(\frac{r}{R}\right)^2 \cos 2\theta + 7\left(\frac{r}{R}\right)^{13} \sin 13\theta$$