NAME ____________________________

(10 pts.) **1.** Determine the values of k, if any, for which the following system has

a) no solution,

b) infinitely many solutions,

c) a unique solution.

$$\begin{aligned} x_1 + 5x_2 + 3x_3 &= 2 \\ 2x_1 + 4x_2 + 5x_3 &= 4 \\ -x_1 + x_2 + -2x_3 &= k \end{aligned}$$

(10 pts.) **2.** Let $\mathbb{A} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 4 & 2 \\ -2 & a & -3 & 3 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ where a is a real number.

a) Calculate $\det \mathbb{A}$ by expanding along row four.

b) Find all a, if any, such that the system $\mathbb{A}\vec{x} = 0$ has a nontrivial (i.e., nonzero) solution.

$\det \mathbb{A} =$

Nontrivial solution if $a =$

(20 pts.) **3. a)** Let

$$\mathbb{A} = \begin{bmatrix} 1 & 1 & -1 & -3 \\ 0 & 1 & 1 & -4 \\ 2 & 3 & -1 & -10 \end{bmatrix}.$$

Find the *Reduced* Row Echelon Matrix for $\mathbb{A}$. What is the rank of $\mathbb{A}$? What is the dimension of the row space of $\mathbb{A}$? What is the dimension of the column space of $\mathbb{A}$?

Rank of $\mathbb{A}$ =

Dimension of the row space =

Dimension of the column space =

b) Now let

$$\mathbb{B} = \begin{bmatrix} 1 & -2 & 0 & -3 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Find a basis for the null space of $\mathbb{B}$, i.e., the set of vectors $\vec{x}$ in $\mathbb{R}^4$ such that $\mathbb{B}\vec{x} = 0$.

(20 pts.) **4.** Diagonalize the symmetric matrix $\mathbb{A} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, i.e., find an *orthogonal* matrix $\mathbb{P}$ such that $\mathbb{P}^{-1}\mathbb{A}\mathbb{P} = \mathbb{D}$, where $\mathbb{D}$ is a diagonal matrix. Find $\mathbb{P}$, $\mathbb{D}$, and $\mathbb{P}^{-1}$.

$\mathbb{P} =$

$\mathbb{D} =$

$\mathbb{P}^{-1} =$

(20 pts.) **5.** The matrix associated to the linear system

$$\begin{aligned} \frac{dx_1}{dt} &= -x_1 + x_2 \\ \frac{dx_2}{dt} &= -x_1 - x_2 \end{aligned}$$

has a complex eigenvalue $\lambda = -1 + i$ with associated complex eigenvector $\vec{a} = \begin{pmatrix} 1 \\ i \end{pmatrix}$. Find a *real valued* general solution to the system. (Do not verify or try to compute λ or $\vec{a}$. Just take them as given and use them.) What is the type of the critical point at the origin? Is it stable, asymptotically stable, or unstable?

Type?

Stability?

(10 pts.) **6.** The origin is a critical point of the non-linear system

$$\frac{dx}{dt} = \sin x + y$$
$$\frac{dy}{dt} = 4x + \sin y$$

Linearize the system at the origin and determine the type and stability of the critical point there.

(10 pts.) **7.** Carefully graph the trajectories in the phase plane for the linear system with general solution

$$c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{3t} + c_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{2t}.$$