NAME ________________________________

Laplace transform information

$$\mathcal{L}(1) = 1/s$$

$$\mathcal{L}(t^n) = n!/s^{n+1}$$

$$\mathcal{L}(e^{at}) = 1/(s-a)$$

$$\mathcal{L}(\sin\omega t) = \omega/(s^2+\omega^2)$$

$$\mathcal{L}(\cos\omega t) = s/(s^2+\omega^2)$$

$$\mathcal{L}(u(t-a)) = e^{-as}/s$$

$$\mathcal{L}(\delta(t-a)) = e^{-as}$$

$$\mathcal{L}(f^{(n)}) = s^n\mathcal{L}(f) - s^{n-1}f(0) - \dots. - f^{(n-1)}(0)$$

$$\mathcal{L}(\int_0^t f(\tau)\,d\tau) = \frac{1}{s}\mathcal{L}(f)$$

$$\mathcal{L}(e^{at}f(t)) = F(s-a)$$

$$\mathcal{L}(u(t-a)f(t-a)) = e^{-as}F(s)$$

$$\mathcal{L}(tf(t)) = -F'(s)$$

$$\mathcal{L}(f(t)/t) = \int_s^\infty F(\sigma)\,d\sigma$$

$$\mathcal{L}(f*g)) = \mathcal{L}(f)\mathcal{L}(g)$$

$$\mathcal{L}(f) = \frac{1}{(1-e^{-ps})}\int_0^p e^{-st}f(t)\,dt \quad \text{if f is p periodic}$$

(10 pts.) **1.** Assume that $y(t)$ is a solution to the problem

$$2y'' + 3y' + y = \delta(t-2) \qquad \text{with } y(0) = 5 \text{ and } y'(0) = 7.$$

Find the Laplace Transform $Y(s)$ of $y(t)$. (DO NOT FIND $y(t)$. Just find $Y(s)$.)

(20 pts.) **2.** a) Graph the function $f(t)$ given below. Next, express $f(t)$ in terms of step functions and simple functions of t.

$$f(t) = \begin{cases} 0 & \text{for } 0 \le t \le 1 \\ (t-1) & \text{for } 1 \le t \le 2 \\ 1 & \text{for } 2 \le t \end{cases}$$

b) Compute the Laplace Transform of $\quad t^2\, u(t-3)$.

(20 pts.) **3.** Find the Inverse Laplace Transforms of the following functions.

a) $\dfrac{3-s}{(s+2)(s+1)}$

b) $\dfrac{e^{-5s}}{(s+1)^2}$

c) $\dfrac{3-s}{(s+1)^2+4}$

(20 pts.) **4.** Find all *positive* eigenvalues for the Sturm-Liouville problem

$$y'' + \lambda y = 0 \qquad \text{with } y'(0) = 0 \text{ and } y(\pi) = 0.$$

(Don't bother checking the $\lambda = 0$ or $\lambda < 0$ cases, and don't find the eigenfunctions.)

(10 pts.) **5.** Compute the Fourier Sine series for $f(x) = \begin{cases} 0 & \text{for } 0 < x < \pi/2 \\ 1 & \text{for } \pi/2 < x < \pi \end{cases}$

(10 pts.) **6.** Show that the functions 1, x^7, and $\cos x$ are orthogonal on the interval $-\pi < x < \pi$ using properties of integrals of odd and even functions when applicable. Given that $f(x) = c_1 \cdot 1 + c_2 x^7 + c_3 \cos x$, express c_2 in terms of integrals over the interval $-\pi < x < \pi$ involving $f(x)$ and the given functions.

(10 pts.) **7.** Let $f(x) = \begin{cases} 0 & \text{for } 0 < x < 1 \\ 1 & \text{for } 1 < x < 3 \\ 0 & \text{for } x > 3 \end{cases}$

a) Find the Fourier Sine Transform of $f(x)$.

b) Let $F(w)$ denote the function you found in part (a) and let g denote the function obtained by taking the Fourier Sine Transform of F. Without calculating anything, explain what the value of $g(2)$ is. What is the value of $g(3)$?