Lecture 1 MA 530 Complex Analysis

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MATH 750
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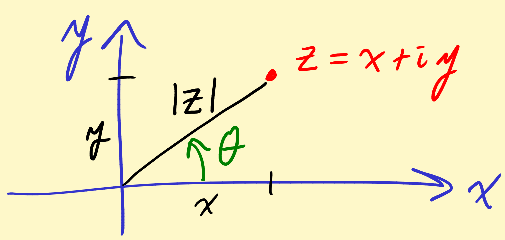
HWK	100 pts
Midterm Exam	100 pts ← Oct 2 in class
2nd Midterm	100 pts ← Take home exam
Final Exam	200 pts
Total	500

complete square

Quadratic formula: $ax^2+bx \underbrace{= a(x-r)^2}$ $= -c$

$-c<0$: Toss in i such that $i^2=-1$.

$\mathbb{C} = \{a+bi \,;\, a,b \in \mathbb{R}\}$. Play games.



$\operatorname{Re} z = x$

$\operatorname{Im} z = y$

$\theta = \operatorname{Arg} z : -\pi < \theta \le \pi$

Principal branch

$|z| = \sqrt{x^2+y^2}$ = "modulus of z"

$\arg z = \{\theta : \theta = \operatorname{Arg} z + 2\pi n \,,\, n \in \mathbb{Z}\}$.

$\arg z$ = "branch of the argument of z"

Polar form of $z = x+iy = r\underbrace{e^{i\theta}}_{\cos\theta + i\sin\theta}$ where $r=|z|$

Calculus game: $e^{(a+bi)} = e^a e^{bi}$

$$= e^a\left(1 + (bi) + \frac{(bi)^2}{2!} + \frac{(bi)^3}{3!} + \cdots\right)$$

$$= e^a\left[\underbrace{\left(1 - \frac{b^2}{2!} + \frac{b^4}{4!} - \cdots\right)}_{\cos b} + i\underbrace{\left(b - \frac{b^3}{3!} + \frac{b^5}{5!} - \cdots\right)}_{\sin b}\right]$$

Def^n: $E(x+iy) = \text{"}e^{x+iy}\text{"} = e^x \cos y + i e^x \sin y$

DeMoivre's formula: $\left(e^{i\theta}\right)^N = e^{iN\theta}$, $N \in \mathbb{Z}$.

Wonderful thing. $1 + \cos\theta + \cos 2\theta + \cdots + \cos N\theta$

$$= \mathrm{Re}\left[1 + e^{i\theta} + e^{i2\theta} + \cdots + e^{iN\theta}\right]$$

$$= \mathrm{Re}\left[1 + \underbrace{(e^{i\theta})}_{r} + (e^{i\theta})^2 + \cdots + (e^{i\theta})^N\right]$$

$$= \mathrm{Re}\left[\frac{1 - (e^{i\theta})^{N+1}}{1 - e^{i\theta}}\right]$$

$$= \mathrm{Re}\left[\frac{1 - [\cos(N+1)\theta + i\sin(N+1)\theta]}{1 - (\cos\theta + i\sin\theta)}\right]$$

= short and clean!

Fourier series: Dirichlet kernel $\sum_{n=-N}^{N} e^{in\theta} =$

$$= 1 + \sum_{n=1}^{N} \left(e^{-i\theta}\right)^n + \sum_{n=1}^{N} \left(e^{i\theta}\right)^n$$

$$= 1 + \left(e^{-i\theta}\right) \underbrace{\sum_{n=0}^{N-1} \left(e^{-i\theta}\right)^n}_{\text{geom ser}} + \left(e^{i\theta}\right) \underbrace{\sum_{n=0}^{N-1} \left(e^{i\theta}\right)^n}$$

$$= \frac{\sin\left(N+\frac{1}{2}\right)\theta}{\sin\frac{\theta}{2}}$$

<u>Algebra games</u> $\quad x+iy \sim (x,y)$

$$(x_1, y_1) + (x_2, y_2) = (x_1+x_2, y_1+y_2)$$

$$(x_1, y_1)\cdot(x_2, y_2) = (x_1x_2 - y_1y_2, x_1y_2 + x_2y_1)$$

$$0 \sim (0,0) \qquad 1 \sim (1,0) \qquad i \sim (0,1)$$

$\mathbb{C} \approx \mathbb{R}^2$ is a <u>field</u>. It is a <u>closed</u> field.

<u>Fund Thm Alg</u> A poly $P(z) = a_N z^N + \cdots a_1 z + a_0$ of deg $N \geq 1$ $(a_N \neq 0)$ has a <u>root</u> $r \in \mathbb{C}$,

$$P(r) = 0.$$

<u>Cor</u> $\quad P(z) = a_N \prod_{n=1}^{N} (z - r_n)$

might repeat

$$= a_N \prod_{k=1}^{m} (z - r_k)^{m_k}, \quad \sum_1^m m_k = N$$

Algebra game :

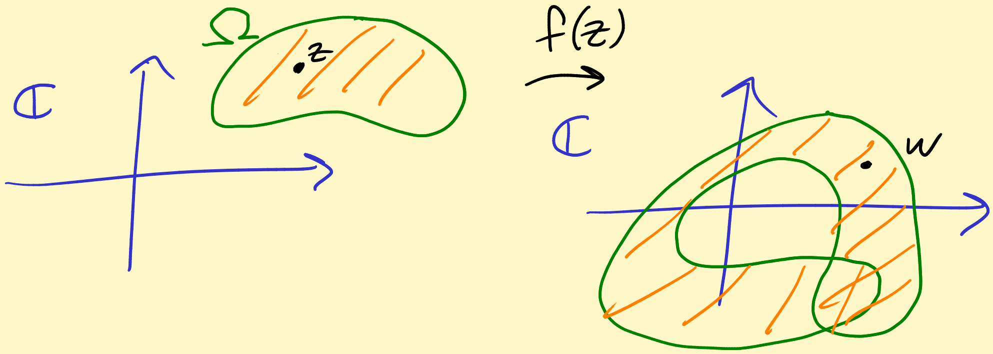
Frobenius Every division ring gotten as a finite extension of $\mathbb{R}$ is either

$$\approx \mathbb{R}$$

$$\approx \mathbb{C}$$

$$\approx \mathbb{H} \Leftarrow \text{Quaternions}$$

MATH 530 :



$w = f(z)$

$u + iv = f(x+iy) \Leftarrow u(x,y) + i\,v(x,y)$

$\mathbb{C} \approx \mathbb{R}^2$ $\qquad f : z \longmapsto w$

$$(x,y) \longmapsto (u(x,y), v(x,y))$$

Complex diff quotient: $DQ = \dfrac{f(z) - f(a)}{z - a}$

MA 530: Complex diff'ble fcns.