Lecture 25 LFTs, Jukovsky map, conformal mapping

$$\frac{z-a}{z-b} \qquad a \mapsto 0, \quad b \mapsto \infty$$

$$\left(\frac{c-b}{c-a}\right)\cdot\frac{z-a}{z-b} \qquad a \mapsto 0, \quad b \mapsto \infty, \quad c \mapsto 1$$

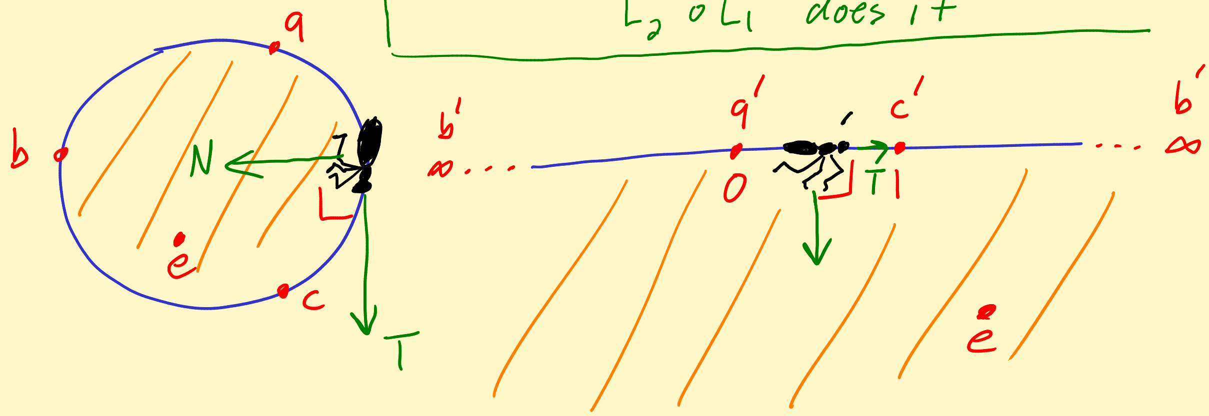
Prob Find LFT: $z_1 \mapsto w_1$, $z_2 \mapsto w_2$, $z_3 \mapsto w_3$ (pts in $\mathbb{C}$)

Sol^n

$$\begin{array}{lcl} & L_1 & & L_2 & \\ z_1 & \mapsto & 0 & \leftarrow\!\shortmid & w_1 \\ z_2 & \mapsto & 1 & \leftarrow\!\shortmid & w_2 \\ z_3 & \mapsto & \infty & \leftarrow\!\shortmid & w_3 \end{array}$$

$L_2^{-1} \circ L_1$ does it

EX

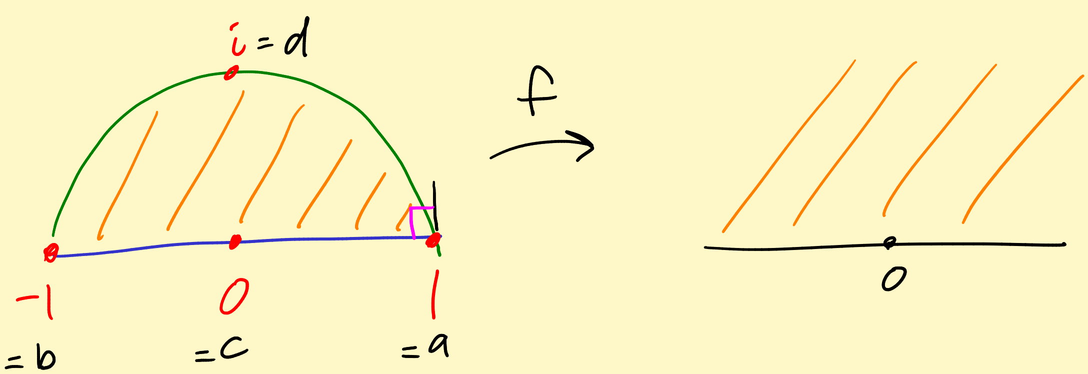


Bug test Use conformality too see which side goes to which side.

One pt test Or, pick a point. See where it goes.

Use fact: $\left\{\begin{array}{c}\text{connected}\\ \text{components of}\\ \mathbb{C} - \{\text{circle}\}\end{array}\right\} \longrightarrow \left\{\begin{array}{c}\text{connected}\\ \text{components of}\\ \mathbb{C} - \{\text{line}\}\end{array}\right\}$

EX

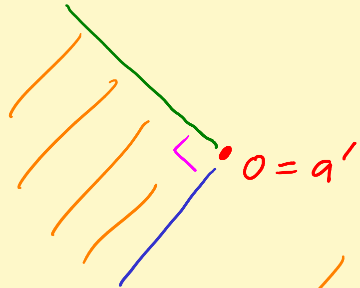


Trick Blow a corner out to pt at ∞ via an LFT.

$$L(z) = \frac{z-1}{z-(-1)} = \frac{z-1}{z+1}$$

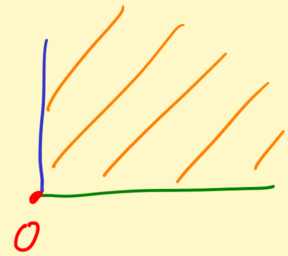
circle, line $\Big\} \to$ lines

Hmmm:

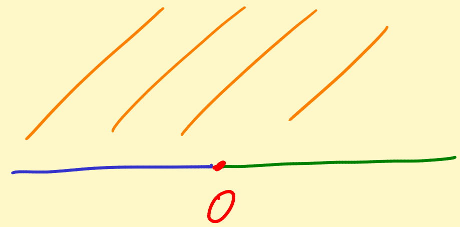


conformal

rotate: $e^{i\theta}z$ $\longrightarrow$



$\xrightarrow{z^2}$



Details

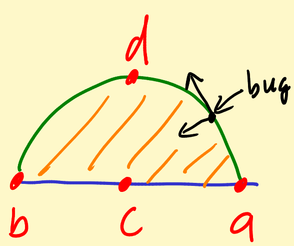
$L(1) = 0$

$L(0) = -1$

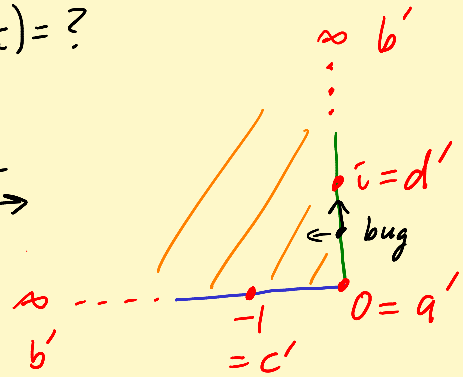
$L(-1) = \infty$

$L(i) = \frac{i-1}{i+1} = i$

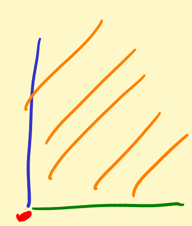
$L(\frac{i}{2}) = ?$



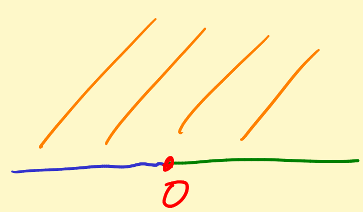
$\xrightarrow{L}$



$\xrightarrow[-iz]{e^{-i\pi/2}z}$

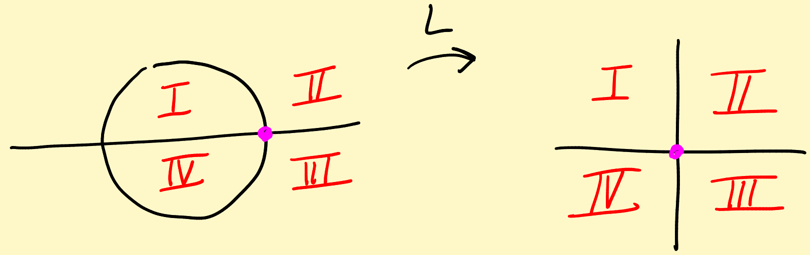


$\xrightarrow{z^2}$

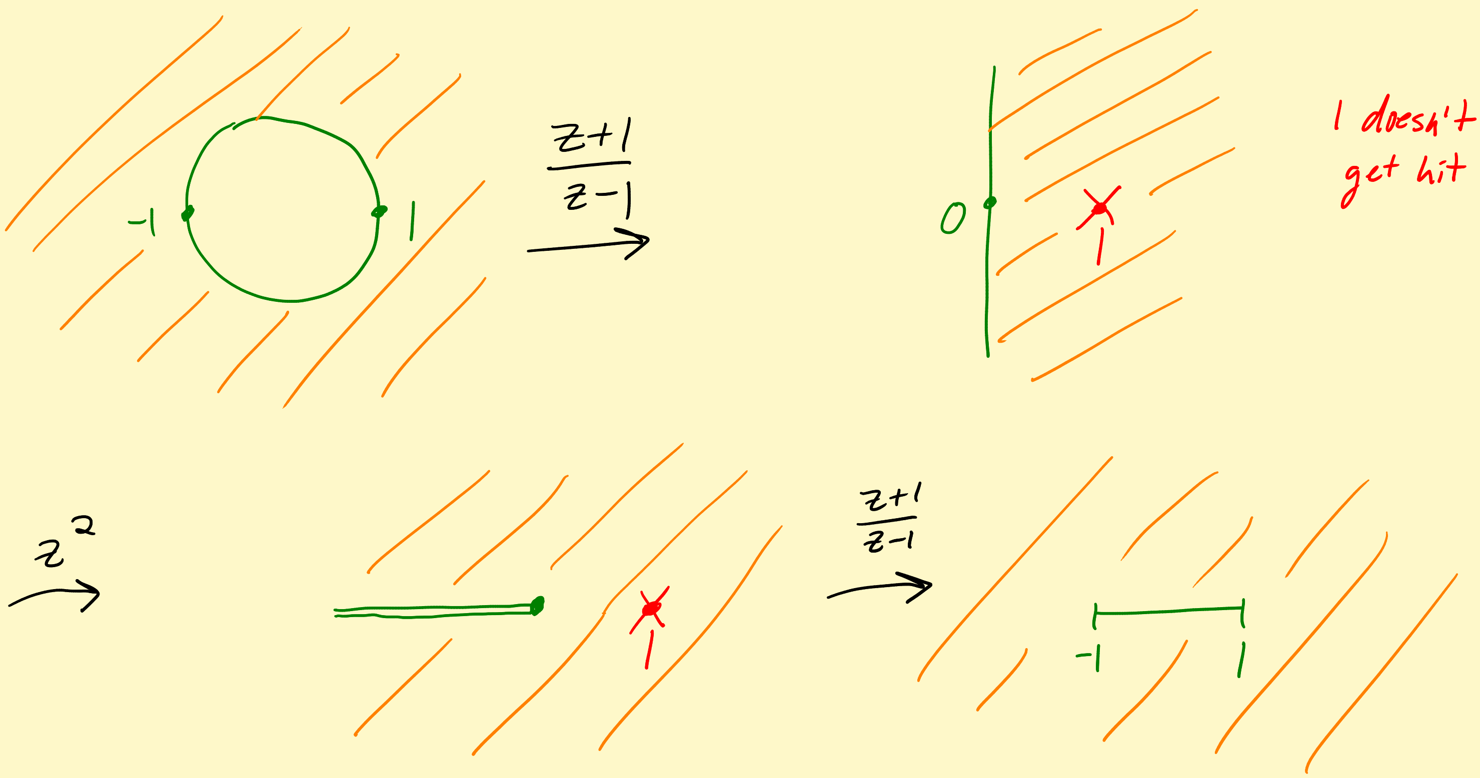


$$f(z) = \left[-i \left(\frac{z-1}{z+1} \right) \right]^2 = -\left(\frac{z-1}{z+1} \right)^2$$

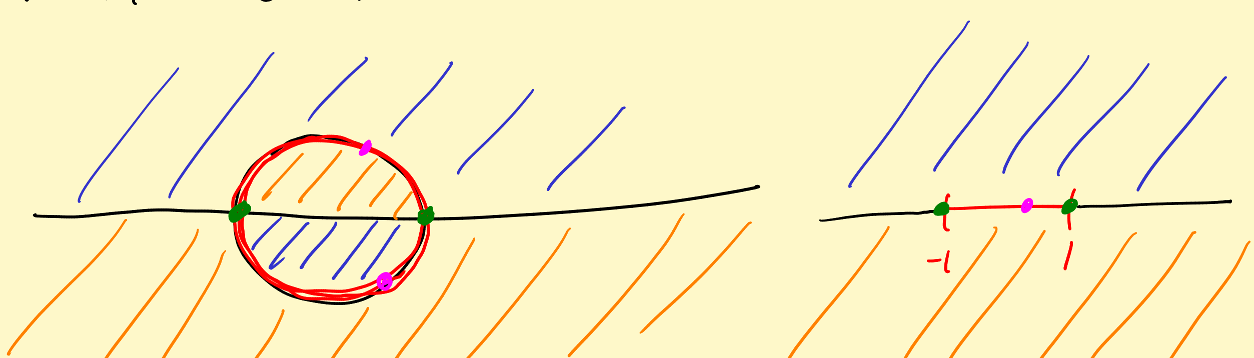
Big picture:



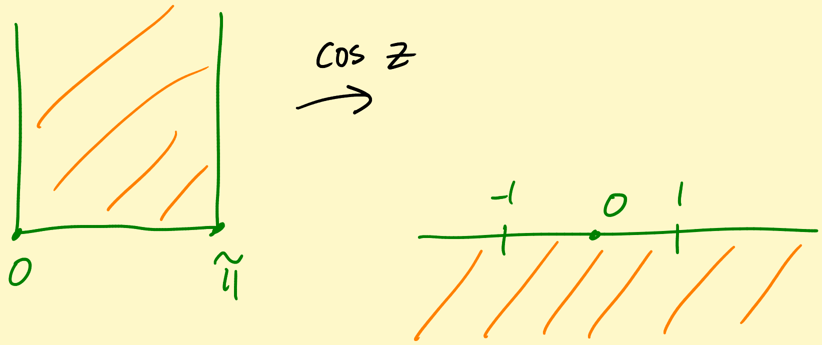
Cook up Jukovsky map $J(z) = \frac{1}{2}\left(z + \frac{1}{z}\right)$

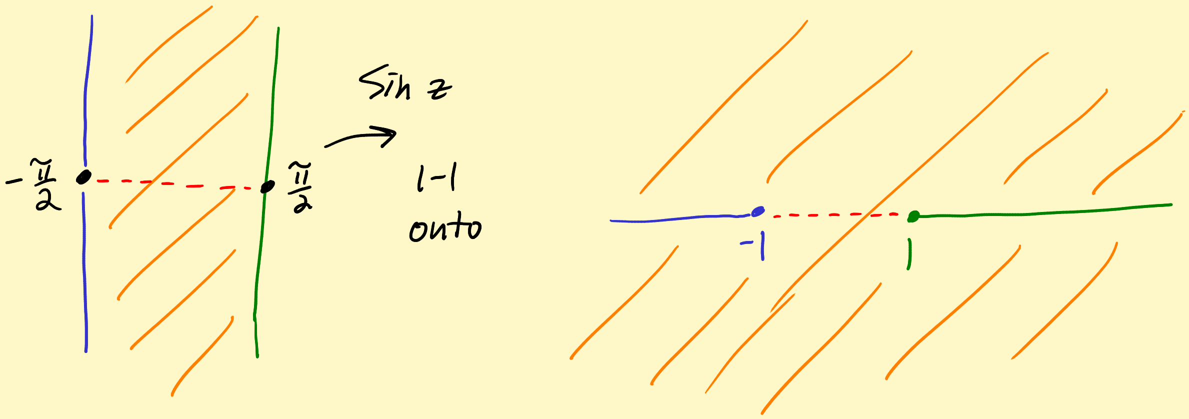


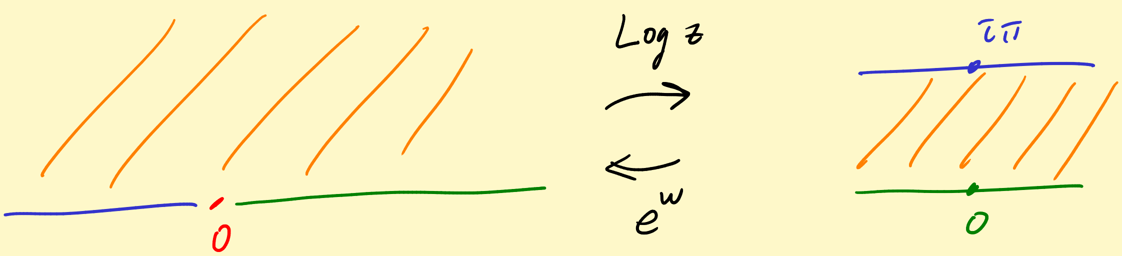
Composition $= J(z)$

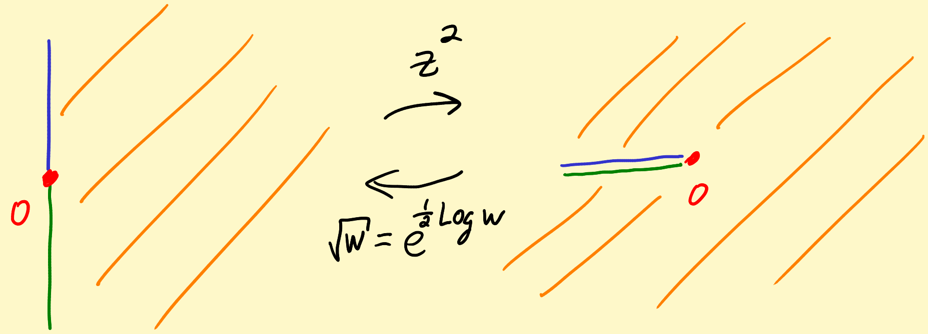


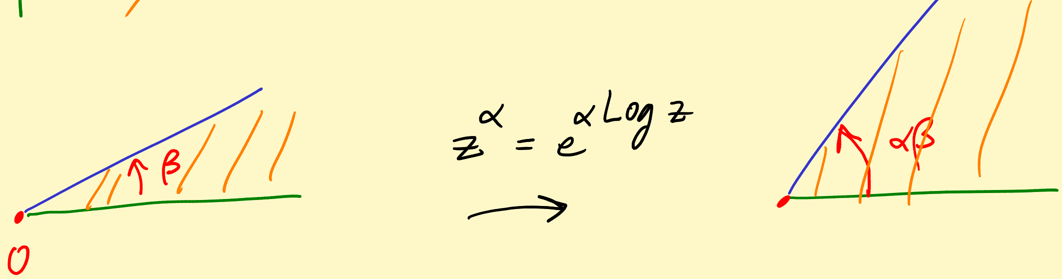
Conformal maps : bag of tricks

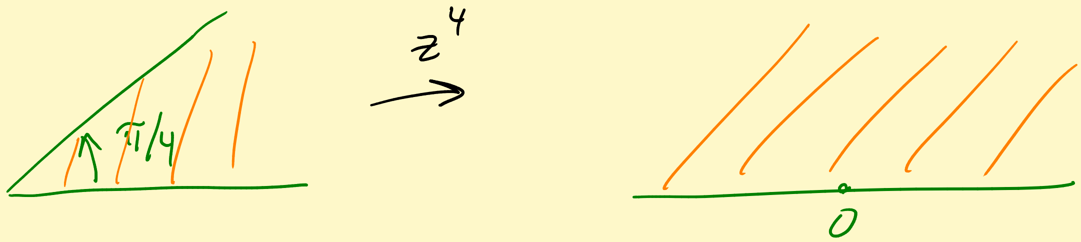




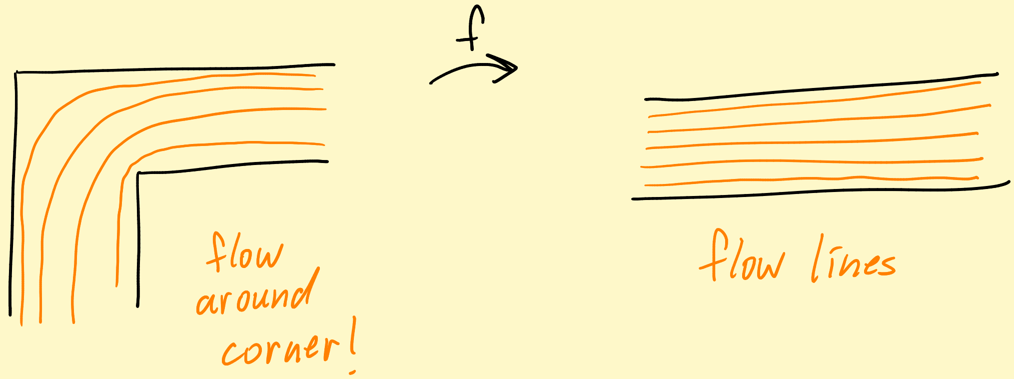








Engineers



Schwarz-Christoffel method