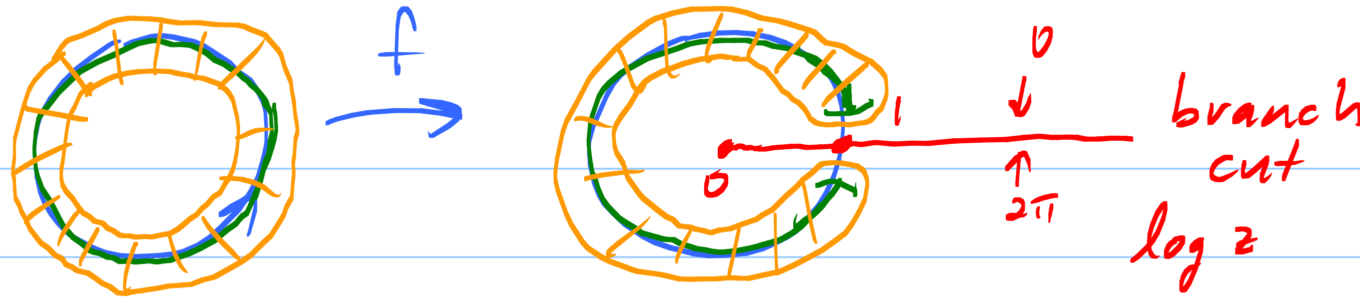
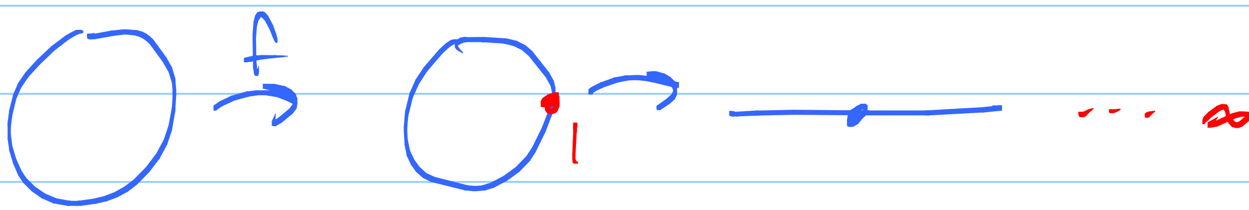


$\#$ zeroes of f inside $C_1(0) = \frac{1}{2\pi i} \int_{C_1(0)} \frac{f'}{f}\, dz$

$\frac{f'}{f} = \frac{d}{dz}(\log f)$ on $\boxed{////}$. So $\int = 0$.

Finally, $|f| = 1$ on $C_1(0) \Rightarrow |f| \leq 1$ on $\overline{D_1(0)}$ by max princ. f has no zeroes. Apply max princ to $1/f$ to see $|f| \geq 1$ on $\overline{D_1(0)}$ too.
So $|f| \equiv 1$. $\Rightarrow f \equiv \lambda$, $|\lambda| = 1$.



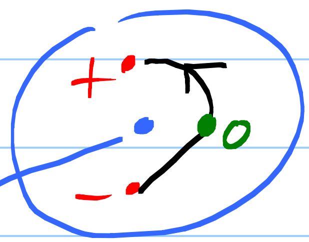
5. u harmonic. How big a set can ∇u vanish on?

Think: $\underbrace{f = u + iv}_{\text{local}}$, $f' = u_x + iv_x = \underbrace{u_x - iu_y}_{\text{analytic on } \Omega}$

$F = u_x - iu_y$ is analytic on Ω.

$\{z : \nabla u = 0\} = \{F = 0\}.$ ← either $\nabla u \equiv 0$ or ∇u has isolated zeroes.

4. Zeroes of harmonic fcns not isolated.



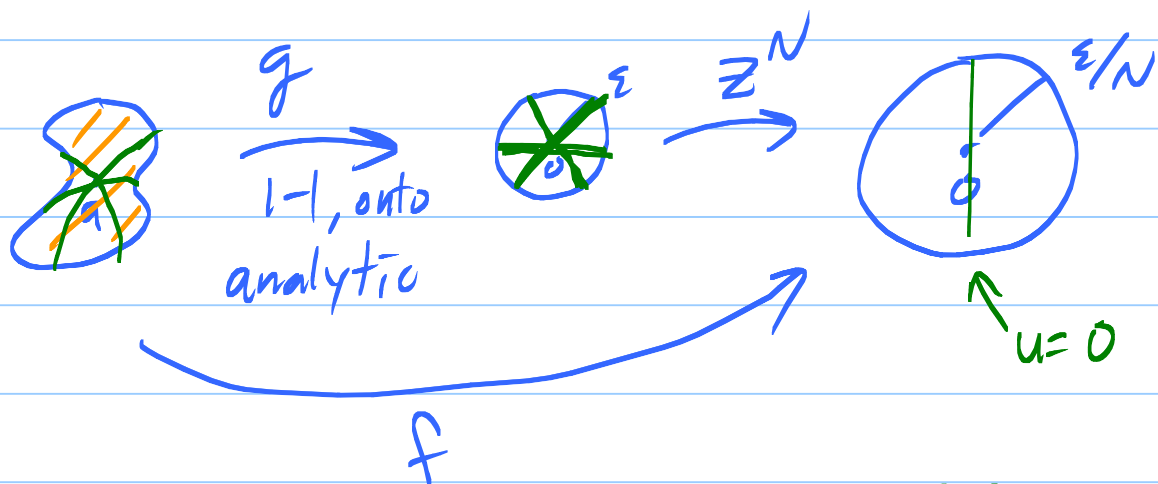
isolated zero

u is always $+$ or always $-$ in $D_\varepsilon(a) - \{a\}$.

But $u(a) = \underbrace{\text{Ave } C_{\varepsilon/2}(a)}_{\text{not } 0!}$ ↯.

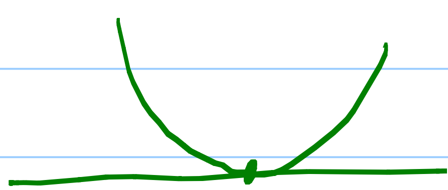
Or local mapping theorem: $f = u + iv$

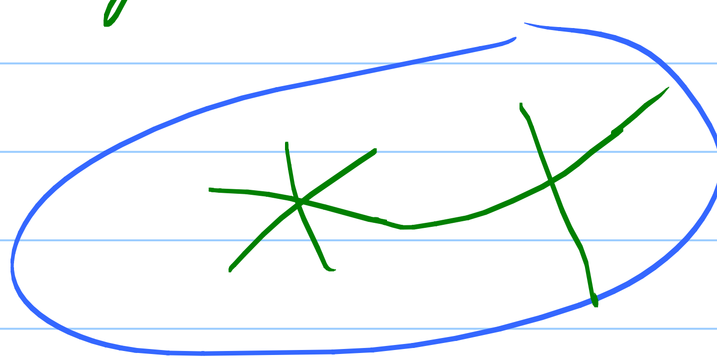
$u(a) = 0$. Can make $v(a)$ too.



→ no $z + w_0$ map

Green curves cross at angle $\frac{2\pi}{N}$.





3. $z^4 - 6z + 3 = 0 \qquad A = \{z : 1 < |z| < 2\}$

big one on $|z|=2$ big one on $|z|=1$

<u>Rouché on $D_2(0)$:</u> $f = z^4, \quad g = z^4 - 6z + 3$

$$|f-g| = |6z-3| \le 6|z| + 3 \le 15 < 16 = |\overbrace{z^4}^{f}|$$

on $|z|=2$ on $|z|=2$

f has 4 zeroes. So does g.

<u>Rouché on $D_1(0)$:</u> $f = -6z, \quad g = z^4 - 6z + 3$

one zero

$$|f-g| = |-z^4-3| \le 1+3 = 4 < 6 = |-6z|$$

on $|z|=1$ on $|z|=1$

So g has <u>one</u> zero in $C_1(0)$.

Finally, g has 3 zeroes in A.