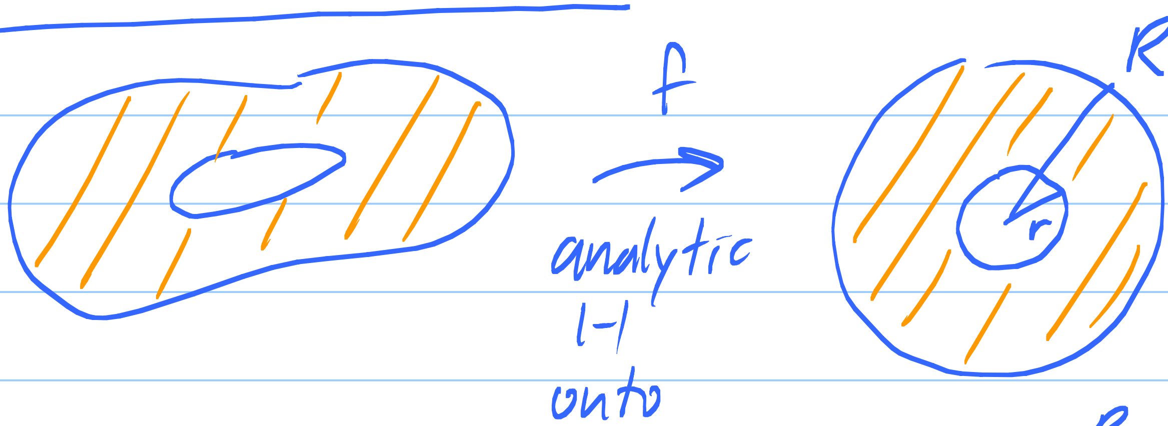
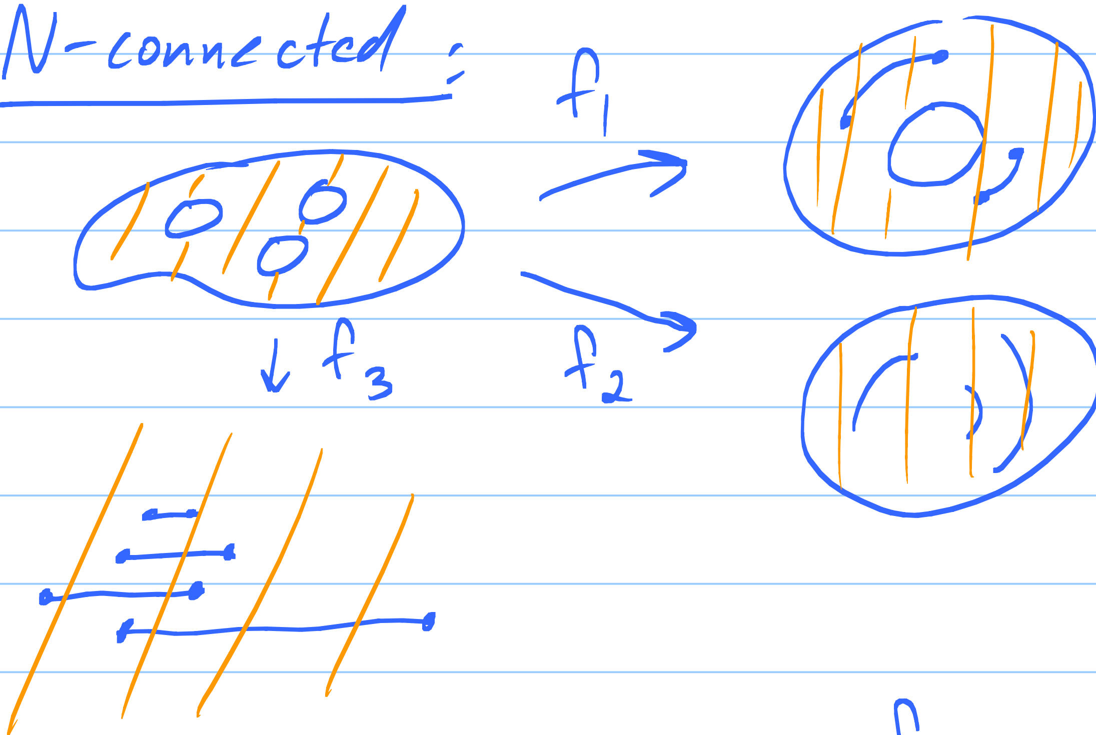
Two connected domains :



Thm: f exists

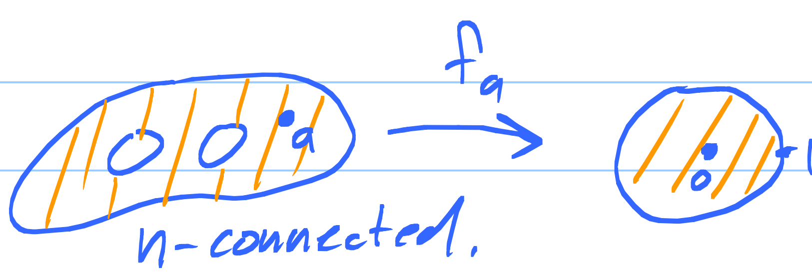
Thm: $\dfrac{r_1}{R_1} = \dfrac{r_2}{R_2}$ ← "modulus"

N-connected :



1-1
onto
analytic

Ahlfors Map :



$f_a'(a)$ is real and max possible.

f_a is analytic and <u>onto</u> and n-to-one (counted with multiplicity). Each boundary curve gets mapped 1-1 onto $C_1(0)$.

<u>Back to harmonic functions:</u>

<u>Problem</u>: Look for a "Cauchy integral formula" for harmonic fcns.

<u>Idea</u>: Suppose u harmonic on $D_r(0)$ with $r>1$. Know $\exists$ harm conv v on $D_r(0)$. Let $f = u + iv$.

$$f(0) = \frac{1}{2\pi}\int_0^{2\pi} f(e^{i\theta})\,d\theta$$

Real part

$$u(0) = \frac{1}{2\pi}\int_0^{2\pi} u(e^{i\theta})\,d\theta$$

ave prop.

Use ave prop <u>plus</u> a Möbius Trans to move points to 0.

Write $\varphi_{-a}(z) = \frac{z+a}{1+\bar{a}z}$ $\quad \varphi_{-a}(0) = a$

$u \circ \varphi_{-a} = \mathrm{Re}[f \circ \varphi_{-a}]$. So $u \circ \varphi_{-a}$ harm.

$$u(a) = u(\varphi_{-a}(0)) = \frac{1}{2\pi}\int_0^{2\pi} u\Big(\underbrace{\frac{e^{i\theta}+a}{1+\bar{a}e^{i\theta}}}_{e^{i\psi}}\Big)\, d\theta$$

Hairy calculus change of variables:

$$= \frac{1}{2\pi}\int_0^{2\pi} u(e^{i\psi}) \left[\underbrace{\frac{1-|a|^2}{|e^{i\psi}-a|^2}\, d\psi}_{d\theta}\right]$$

Poisson Integral Formula: (p.66: 11. in Stein).

u harmonic on $D_r(0)$, $r>1$.

$$u(a) = \int_0^{2\pi} P(a,\psi)\, u(e^{i\psi})\, d\psi$$

where $P(a,\psi) = \frac{1}{2\pi}\frac{1-|a|^2}{|e^{i\psi}-a|^2}$ is

the Poisson kernel.

Aha! Bet we can use this to solve the Dirichlet Prob.

Trick: Apply formula to $u(z) = u(\rho z)$, $\rho<1$.
Let $\rho \nearrow 1$.