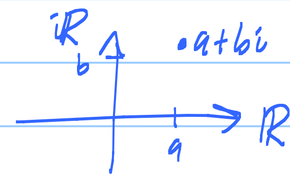
Lecture 1 MA 530 Steve Bell MATH 750 494-1497 bell@purdue.edu

Grader: Seongjun Choi

HWK	100
1 midterm	100
1 FE	200
	400

$\mathbb{C} \approx \mathbb{R}^2$ $\quad i^2 = -1$



$x^2 + 1 = 0$

$(a_1, b_1) + (a_2, b_2) = (a_1 + a_2, b_1 + b_2)$

$(a_1, b_1) \cdot (a_2, b_2) = (a_1 a_2 - b_1 b_2, a_1 b_2 + a_2 b_1)$

$\mathbb{C} \approx \mathbb{R}^2$ is a field

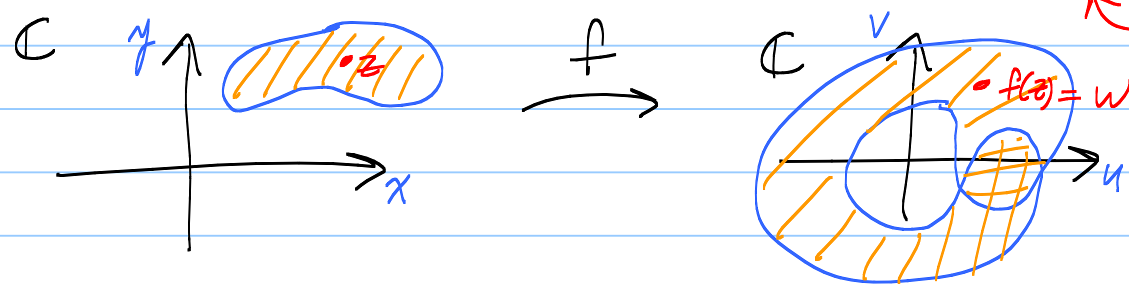
$(0,0) \sim 0 \qquad (1,0) \sim 1 \qquad (0,1) \sim i$

Frobenius: Every division ring that is a finite extension of $\mathbb{R}$ is either

1) $\approx \mathbb{R}$
2) $\approx \mathbb{C}$
3) $\approx$ Quaternions

Fund. Thm. Algebra: Given a complex poly $p(z) = a_N z^N + \cdots + a_1 z + a_0$ of degree $N \geq 1$, then $p(z)$ has a root in $\mathbb{C}$, i.e., $\exists z_0 \in \mathbb{C}$ with $p(z_0) = 0$. Consequently $p(z) = a_N \prod_{k=1}^{N} (z - z_k)$

z_k's can repeat



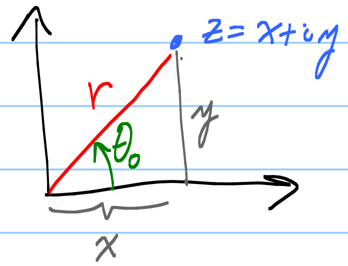
$w = f(z)$

$u + iv = f(x + iy)$

$u(x,y) \quad v(x,y)$

$(x, y) \longmapsto (u(x,y), v(x,y))$

Ex: $f(z) = z^2 = (x+iy)^2 = \underbrace{(x^2 - y^2)}_{u(x,y)} + i\,\underbrace{2xy}_{v(x,y)}$



$x = \operatorname{Re} z$

$y = \operatorname{Im} z$

$r = |z| = \sqrt{x^2+y^2}$ "modulus of z"

$\theta_0 = \operatorname{Arg} z$ = angle $-\pi < \theta_0 \le \pi \leftarrow$ Principal arguement

$\arg z = \{\theta_0 + 2\pi n : n \in \mathbb{Z}\} \leftarrow$ set

Def^n: $e^{i\theta} = \cos\theta + i\sin\theta \leftarrow$ Def^n

$$e^{x+iy} = e^x e^{iy} = (e^x \cos y) + i\,(e^x \sin y) \leftarrow \text{Def}^n$$

$(e^{i\theta})^2 = e^{i2\theta} \leftarrow$ Trig identities

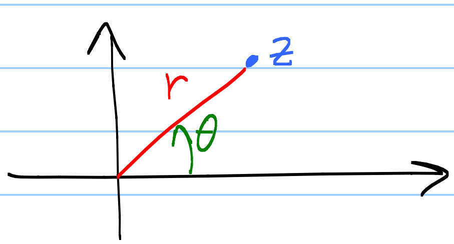
$(e^{i\theta})^N = e^{iN\theta} \leftarrow$ de Moivre's

Fun Thing: $1 + \sin\theta + \sin 2\theta + \cdots + \sin N\theta$

$$1 + \operatorname{Im}\left[1 + e^{i\theta} + e^{i2\theta} + \cdots + e^{iN\theta}\right]$$

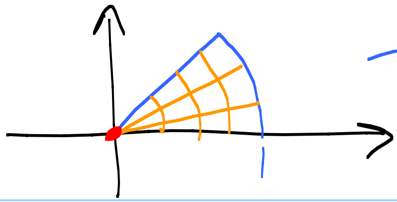
$$1 + \operatorname{Im}\left[1 + (e^{i\theta}) + (e^{i\theta})^2 + \cdots + (e^{i\theta})^N\right]$$

$$1 + \operatorname{Im}\left[\frac{1-(e^{i\theta})^{N+1}}{1-e^{i\theta}}\right]$$ (Σ geom series)

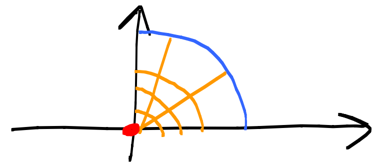


$z = (r\cos\theta) + i(r\sin\theta) = re^{i\theta}$

= "polar form"



$z^2 = (re^{i\theta})^2 = r^2 e^{i2\theta}$



Topology on $\mathbb{C}$: $\approx \mathbb{R}^2$ $\quad d(z_1, z_2) = |z_1 - z_2|$ ← dist fcn

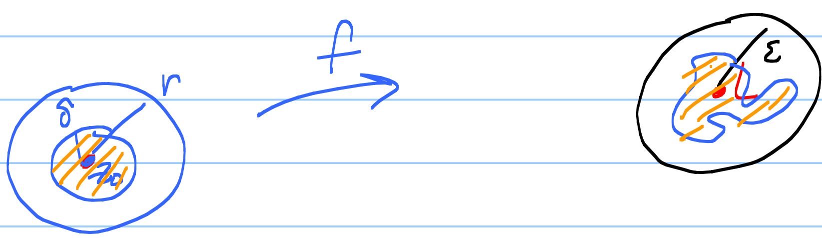
$D_r(z_0) = \{z : |z - z_0| < r\}$ ← disc of radius r about z_0

Def^n: $S \subset \mathbb{C}$ is open, if, given $z_0 \in S$, there is a disc $D_r(z_0) \subset S$, $(r > 0)$.

EX:  $= D_1(0) \cap \{\mathrm{Im} > 0\}$

Def^n: Suppose f is a $\mathbb{C}$-valued fcn on $D_r(z_0) - \{z_0\}$.

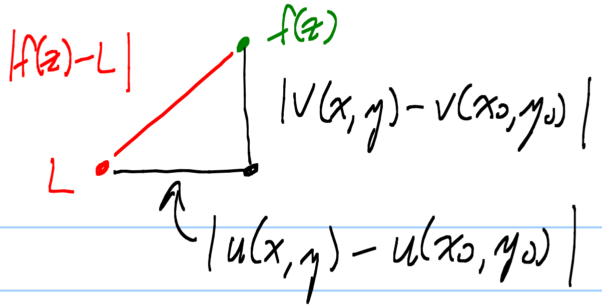
$\lim_{z \to z_0} f(z) = L$ means, given $\varepsilon > 0$, there is a $\delta > 0$ such that $|f(z) - L| < \varepsilon$ when $|z - z_0| < \delta$ and $z \neq z_0$.



Fact: Basic limit laws $\mathbb{R} \to \mathbb{R}$ hold $\mathbb{C} \to \mathbb{C}$, same proofs.

Baby Theorem: $u + iv = f(x + iy)$ $\quad z_0 = x_0 + iy_0$

$$\lim_{z \to z_0} f(z) = L \iff \begin{cases} \lim_{(x,y) \to (x_0,y_0)} u(x,y) = \mathrm{Re}\, L \\ \lim_{(x,y) \to (x_0,y_0)} v(x,y) = \mathrm{Im}\, L \end{cases}$$



<u>Complex diff'ble</u>: Ex $f(z) = z^2$

$$DQ = \frac{f(z)-f(a)}{z-a} = \frac{z^2-a^2}{z-a} = \frac{(z-a)(z+a)}{z-a} = z+a$$

$$\longrightarrow a+a = 2a \quad \text{as} \quad z \to a.$$

<u>Next time</u>: $\bar{z}$ is <u>not</u> complex diff.