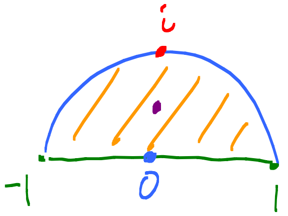
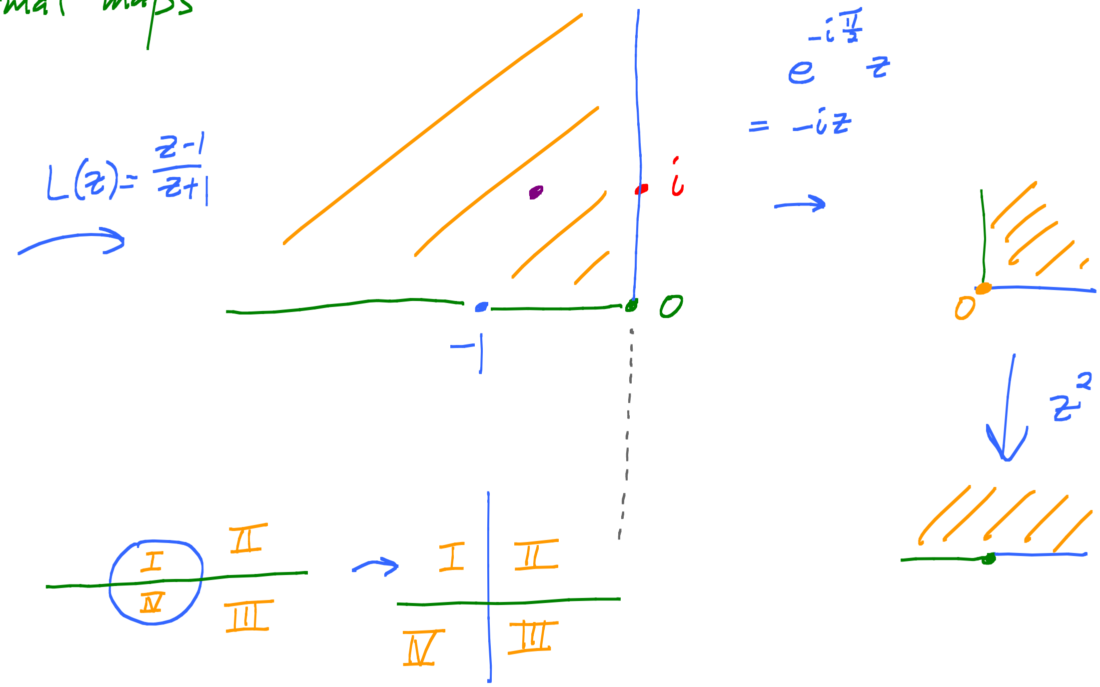
Lecture 24 Conformal maps





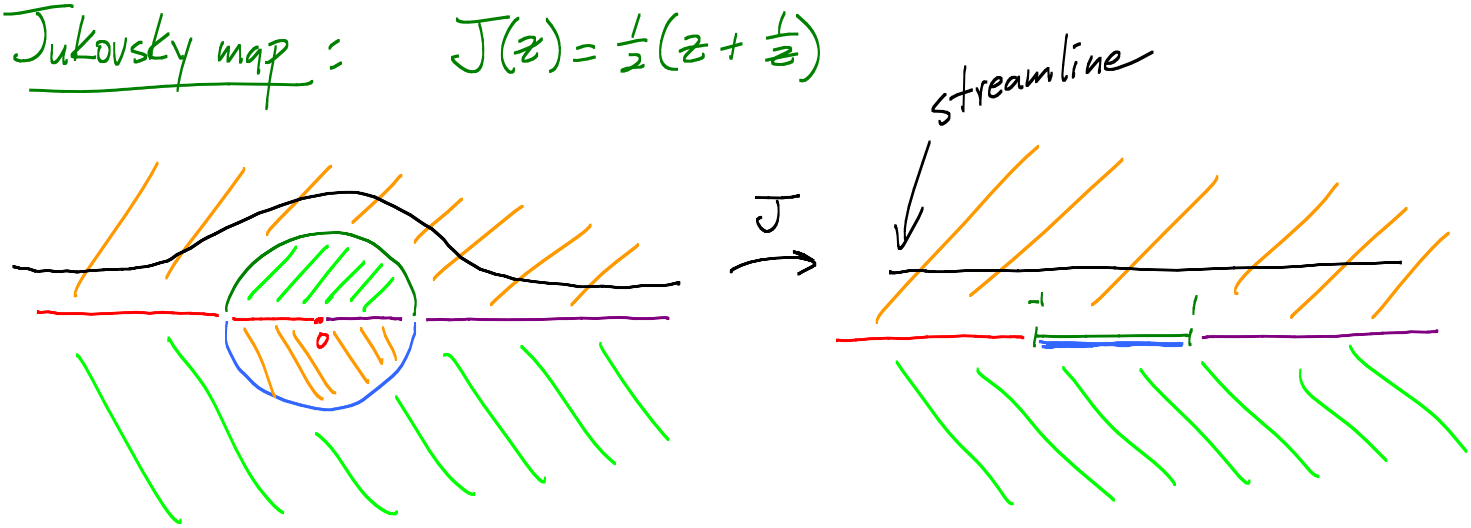
$L(0) = -1$

$L(1) = 0$

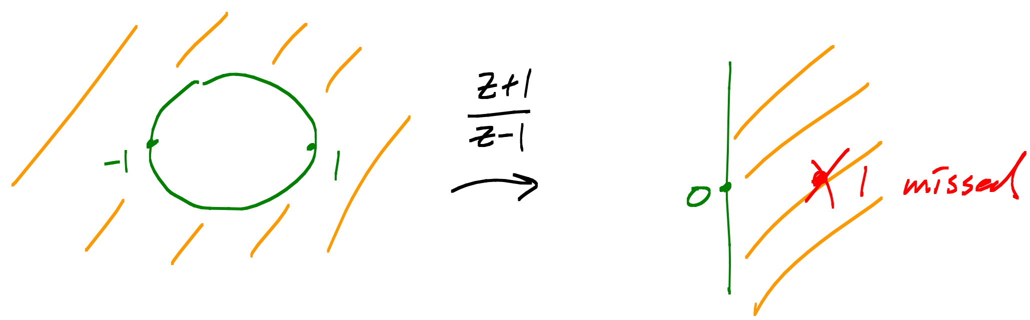
$L(-1) = \infty$

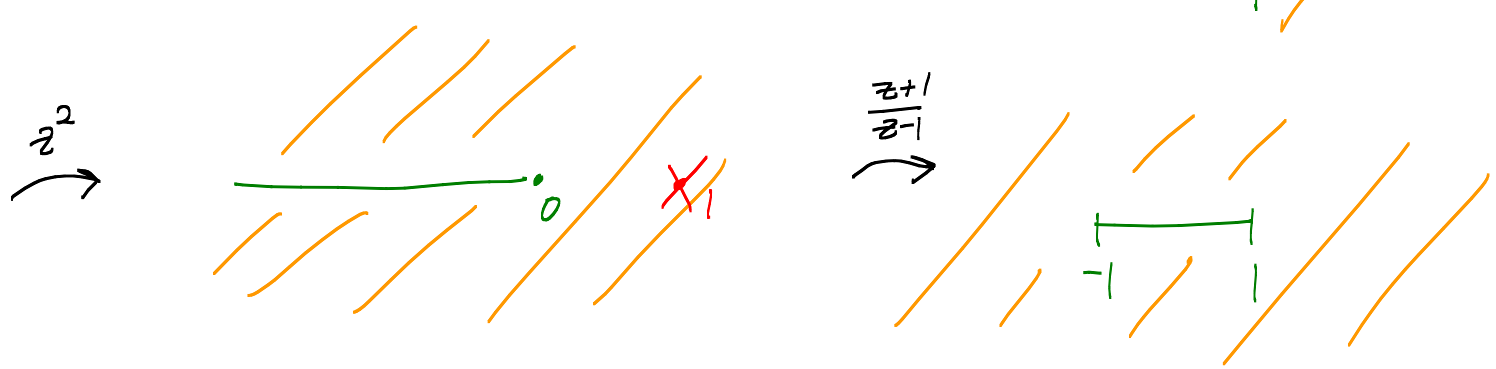
$L(i) = \frac{i-1}{i+1} = i$

Jukovsky map: $J(z) = \frac{1}{2}\left(z + \frac{1}{z}\right)$



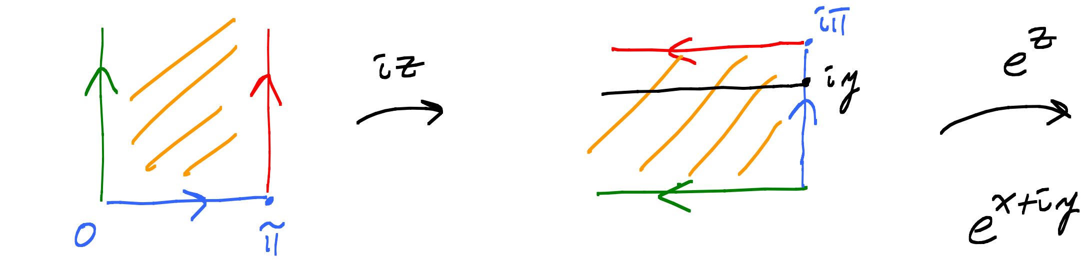
Cooking up J:





Composition $= J(z)$!

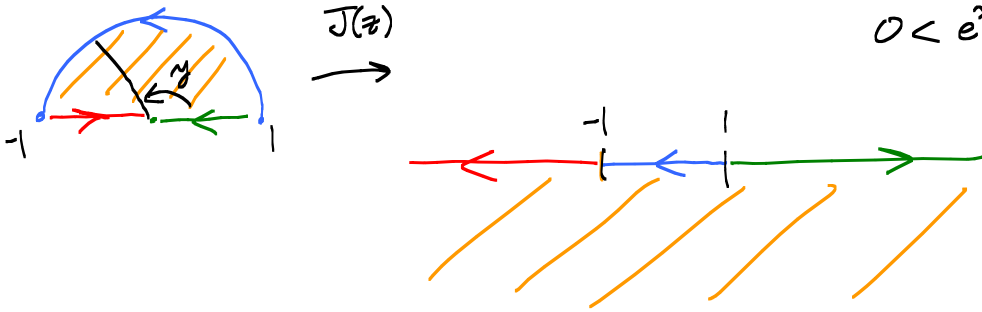
Complex cosine: $\cos z = \dfrac{e^{iz}+e^{-iz}}{2} = \dfrac{1}{2}\left(e^{iz}+\dfrac{1}{e^{iz}}\right) = J(e^{iz})$



$e^{x+iy} = e^x e^{iy}$

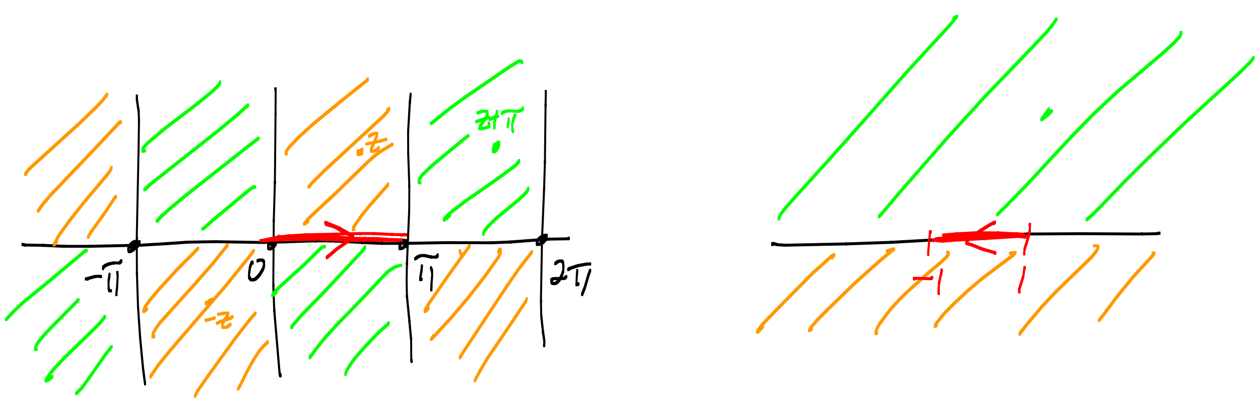
$-\infty < x < 0$

$0 < e^x < 1$

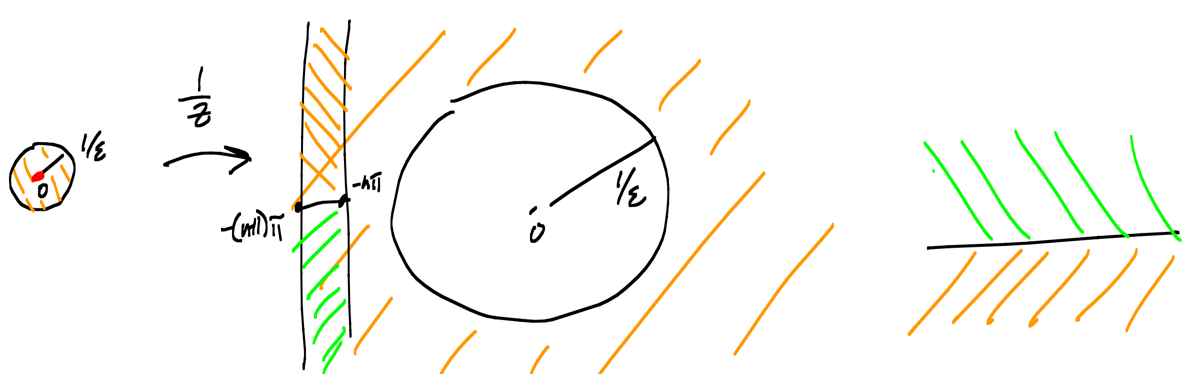


But $\cos(-z) = \cos z$

$\cos(z+\pi) = -\cos z$



See $\cos\left(\frac{1}{z}\right)$ has an essential sing at 0. ($\cos z$ has an ess. sing at ∞.)



$\cos\left(\frac{1}{z}\right)$ covers all of $\mathbb{C}$ ∞ many times. ✓

Branches of $\cos^{-1} w$:

$$w = \frac{1}{2}\left(e^{iz} + \frac{1}{e^{iz}}\right) \quad \cdot e^{iz}$$

$$2e^{iz} w = (e^{iz})^2 + 1$$

$$0 = (e^{iz})^2 - 2w(e^{iz}) + 1$$

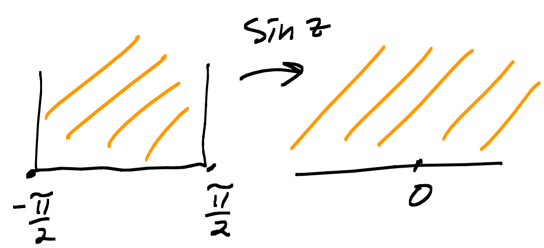
$$e^{iz} = \frac{2w + \sqrt{4w^2 - 4}}{2}$$

$$z = \frac{1}{i} \log\left(w + \sqrt{w^2 - 1}\right)$$

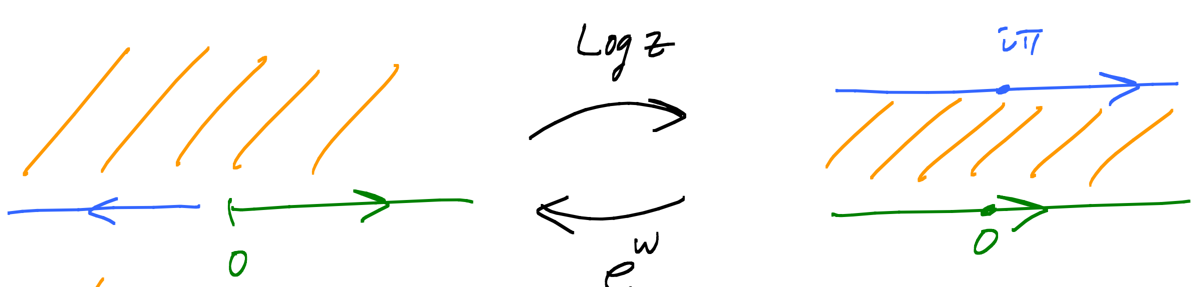
Liousville: e^z and its inverse plus algebraic fcns generate the "elementary fcns."

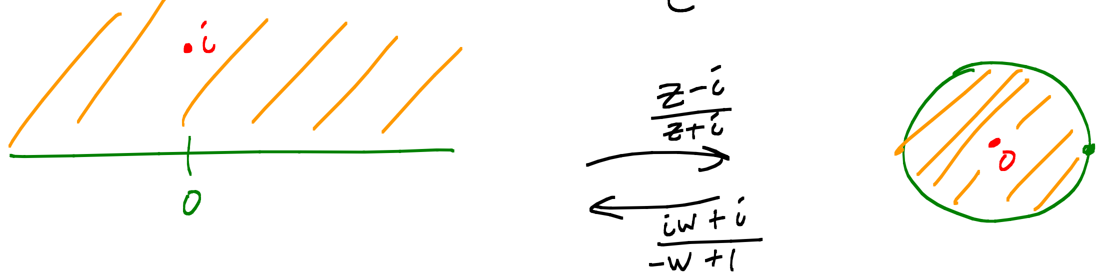
Thm: e^{x^2} does not have an antiderivative that is an elem fcn.

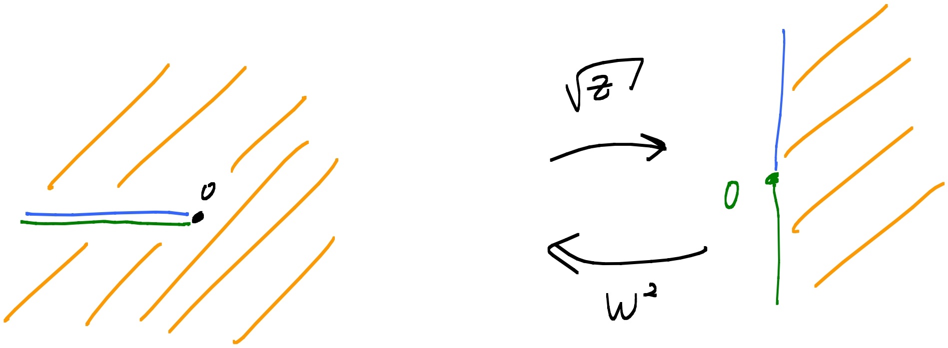
Remark: $\sin(z) = \cos(z - \frac{\pi}{2})$

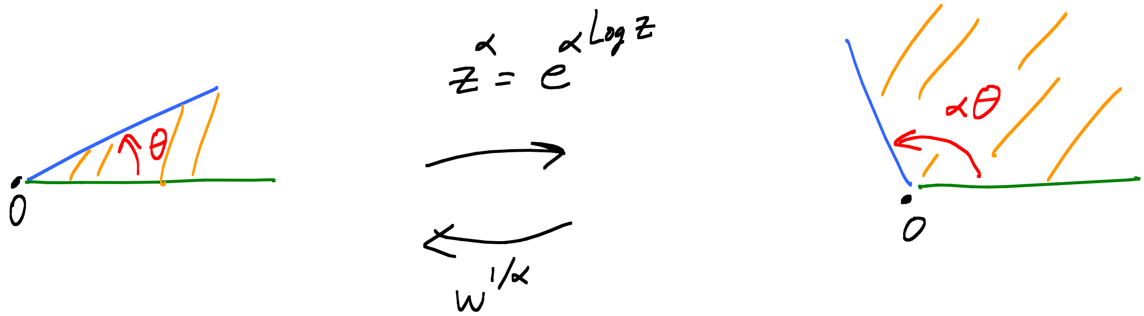


Other useful maps:

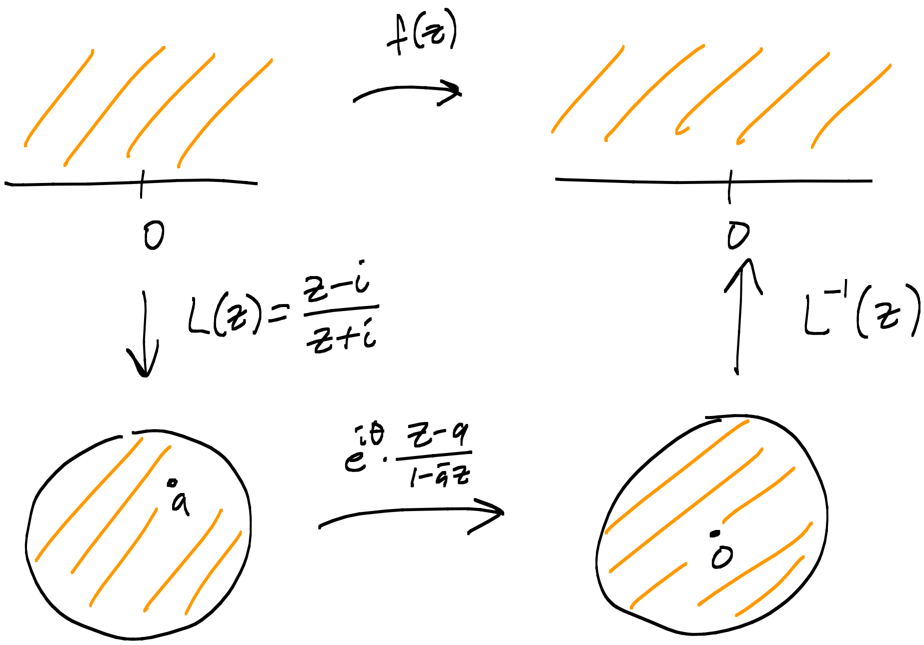








Prob: What is Aut(UHP)?



Stein, Chap. 8