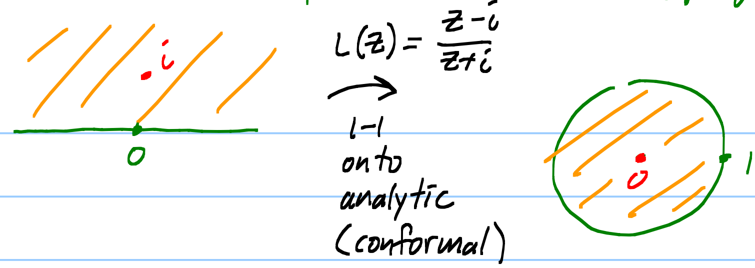
Lecture 24 LFTs, Jukovsky map, conformal mapping

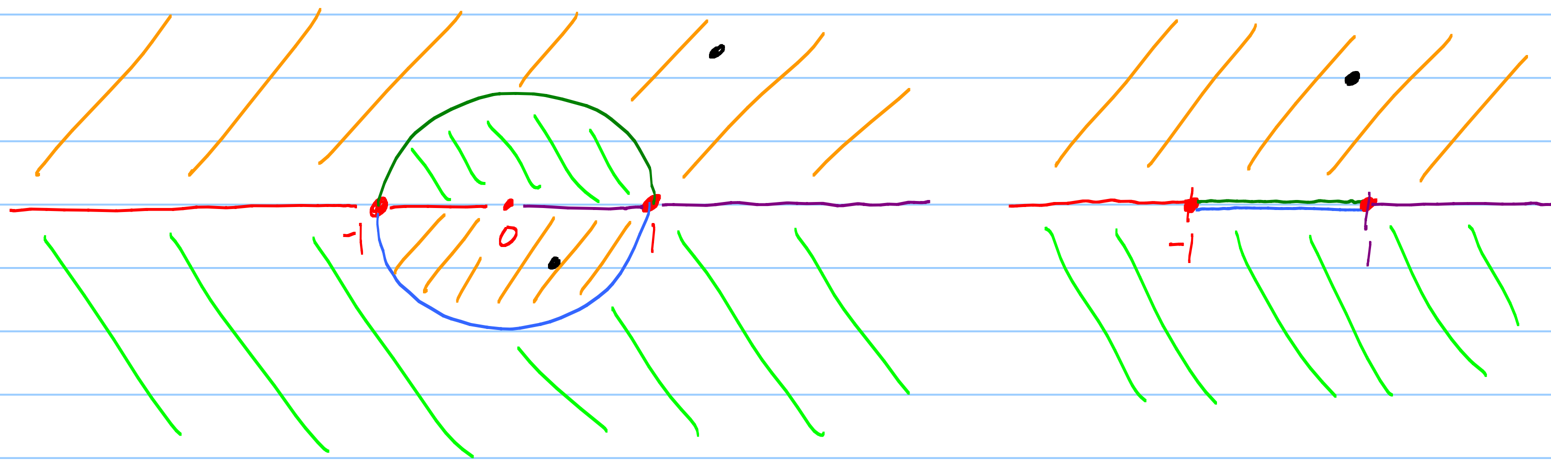
HWK 6 due Thurs

Favorite LFT :



Jukovsky map

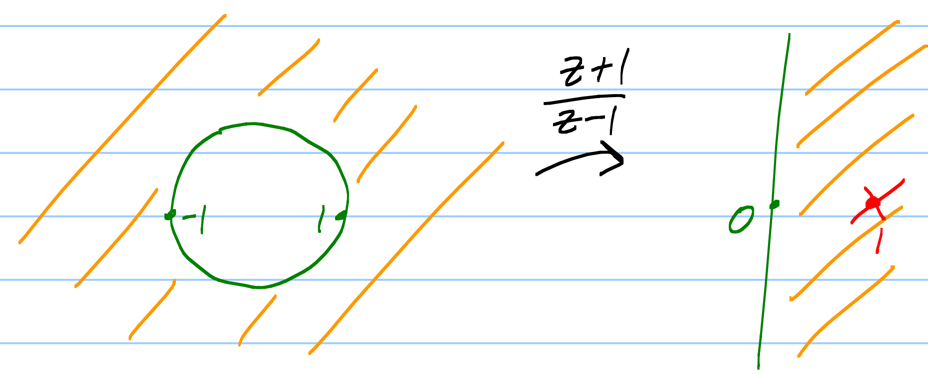
$J(z) = \frac{1}{2}\left(z + \frac{1}{z}\right)$

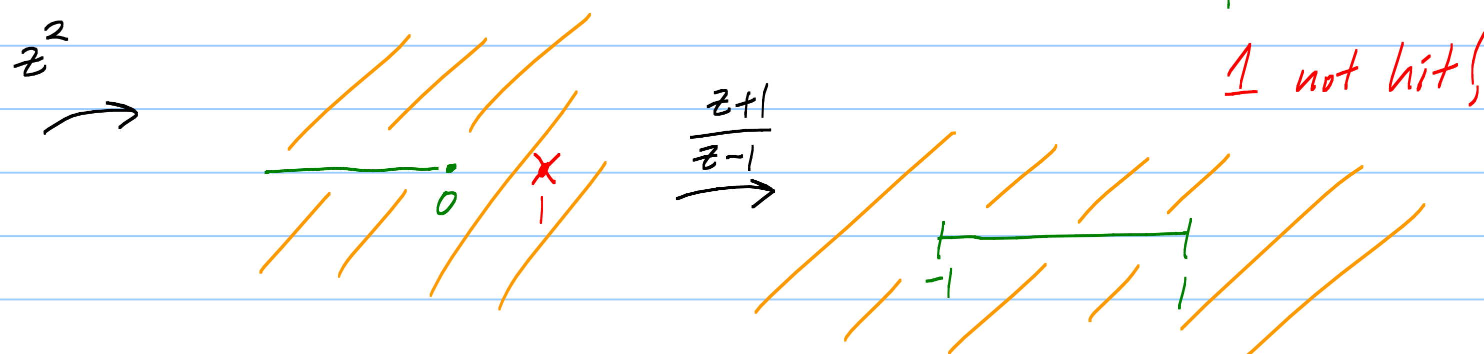


$J'(\pm 1) = 0$ $\quad J: -1 \longmapsto -1$ with mult two

$\qquad\quad 1 \longmapsto 1$ " " "

Cook up $J(z)$:

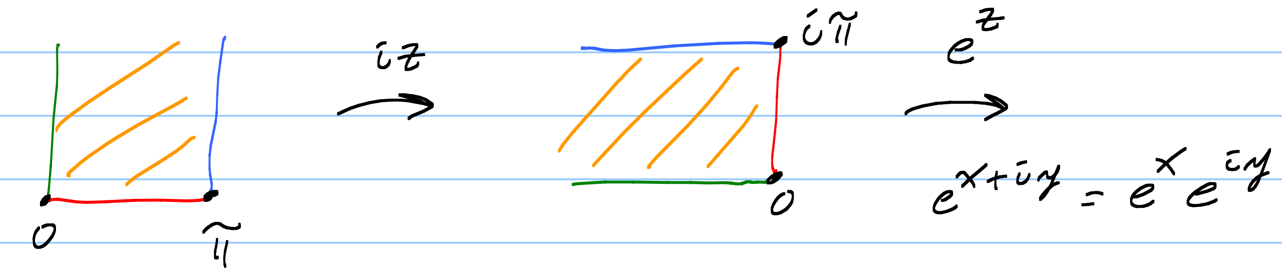




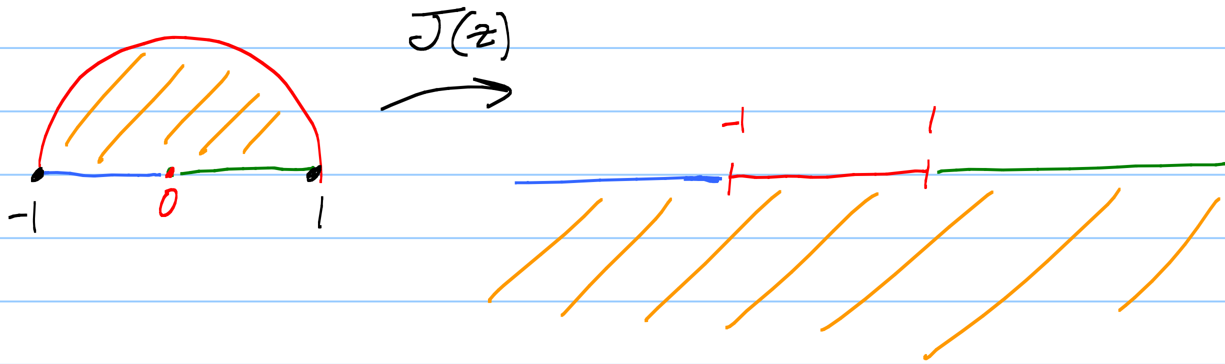
Composition $= J(z)$!

Complex cosine

$$\cos z = \frac{e^{iz} + e^{-iz}}{2} = J(e^{iz})$$

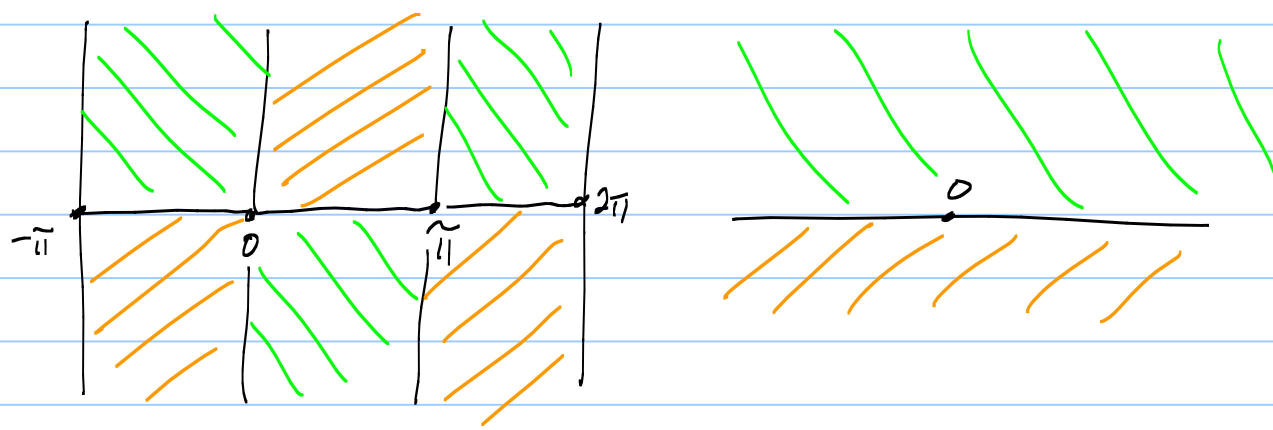


$$e^{x+iy} = e^x e^{iy}$$



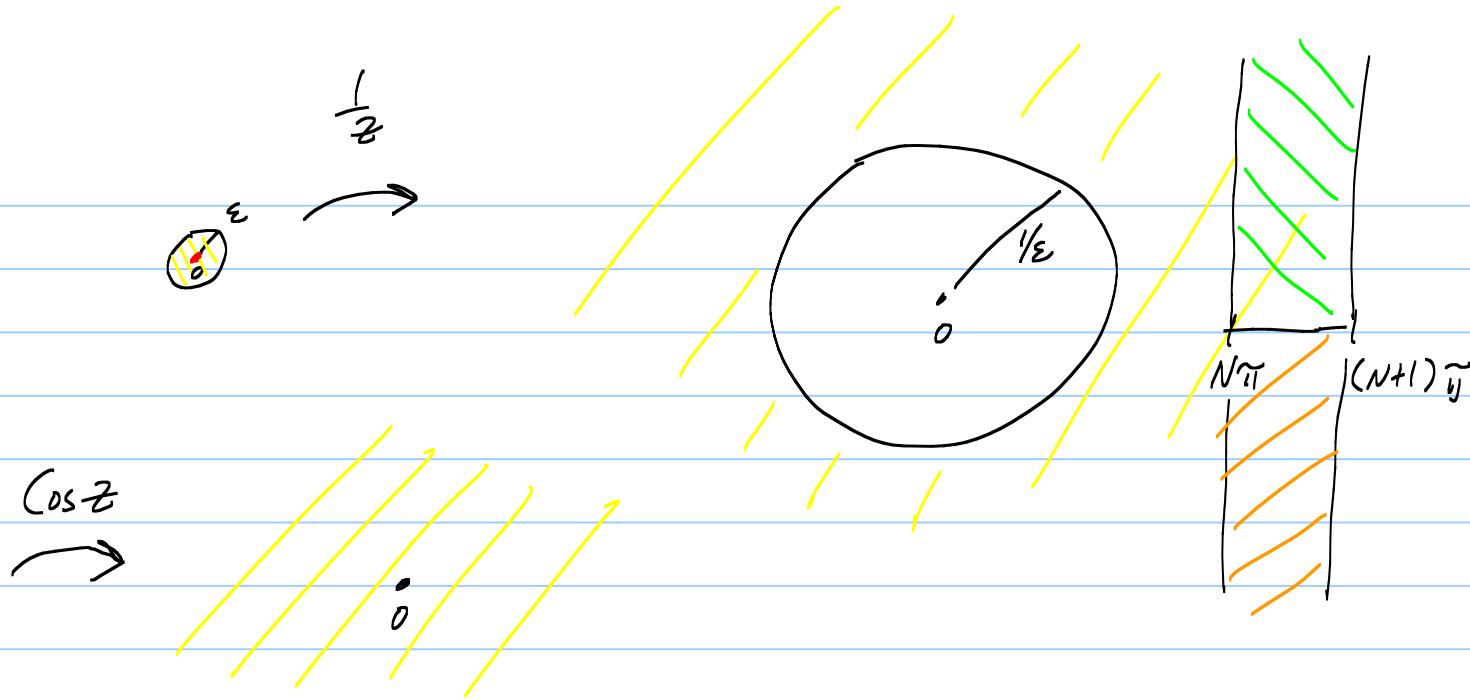
Composition $= \cos z$

$$\cos(-z) = \cos z$$
$$\cos(z+\pi) = -\cos z$$



See $\cos\frac{1}{z}$ maps $\mathbb{C} \to \mathbb{C}$ infinite to one, missing nothing.

$\cos z$ has an "essential singularity at ∞," meaning that $\cos\frac{1}{z}$ has an essential sing at 0.



$f(z) = \cos\frac{1}{z}$ $\quad$ $f(D_\varepsilon(0) - \{0\})$ is dense in $\mathbb{C}$, no matter how small $\varepsilon > 0$.

Branches of $\cos^{-1} w$

$$w = \cos z = \frac{1}{2}\left(e^{iz} + e^{-iz}\right)$$

Solve for z: Mult by $2e^{iz}$: $\quad 2e^{iz} w = \left(e^{iz}\right)^2 + 1$

$$0 = \left(e^{iz}\right)^2 - 2w\left(e^{iz}\right) + 1$$

$\leftarrow$ pick correct $\sqrt{}$

$$e^{iz} = \frac{2w + \sqrt{4w^2 - 4}}{2} = w + \sqrt{w^2 - 1}$$

$$iz = \log\left(w + \sqrt{w^2 - 1}\right)$$

$$z = \frac{1}{i}\log\left(w + \sqrt{w^2 - 1}\right) \leftarrow \cos^{-1} w$$

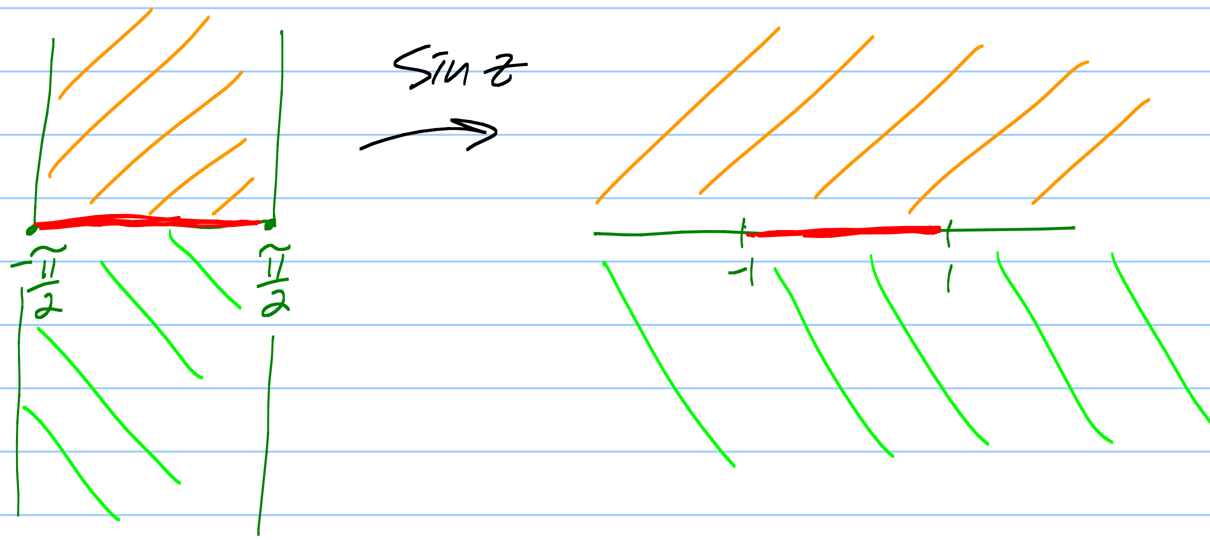
<u>Liouville</u>: e^z and its inverse plus algebraic functions generate the "elemtary fcns".

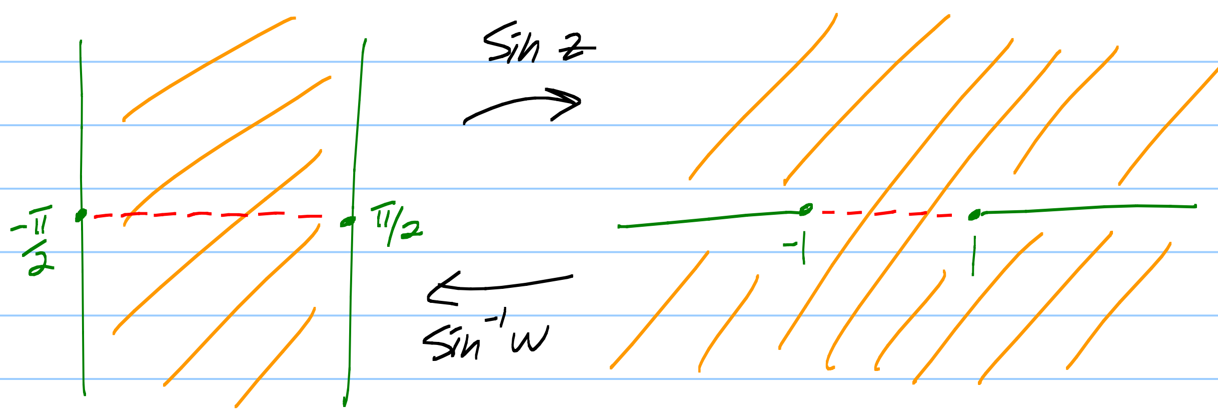
Thm: e^{x^2} does <u>not</u> have an antiderivative that is an elem fcn!

Alg fcns: $P(z,w) = \sum_{n=0}^{N}\sum_{m=0}^{M} c_{nm} z^n w^m \equiv 0$

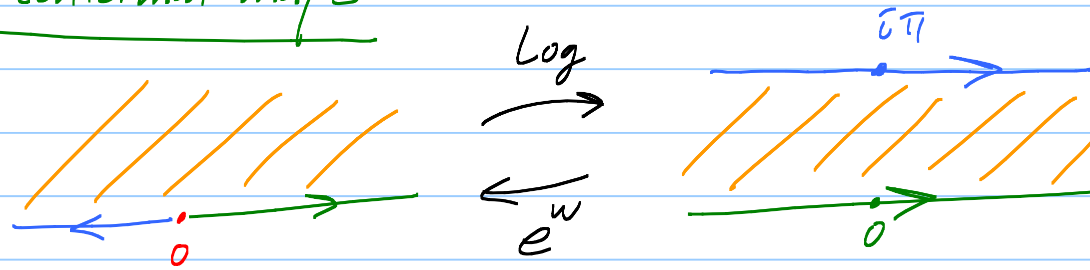
defines $w =$ fcn of z locally away from singular points.

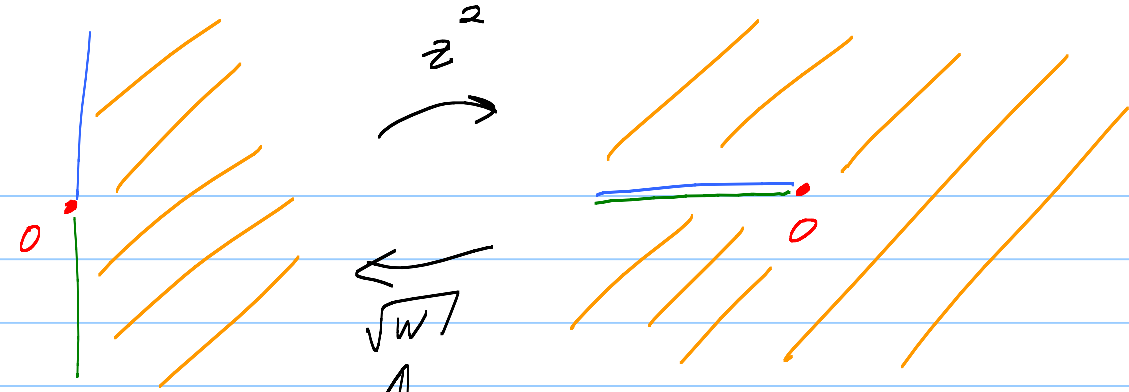
Remark $\sin z = \cos(z - \frac{\pi}{2})$





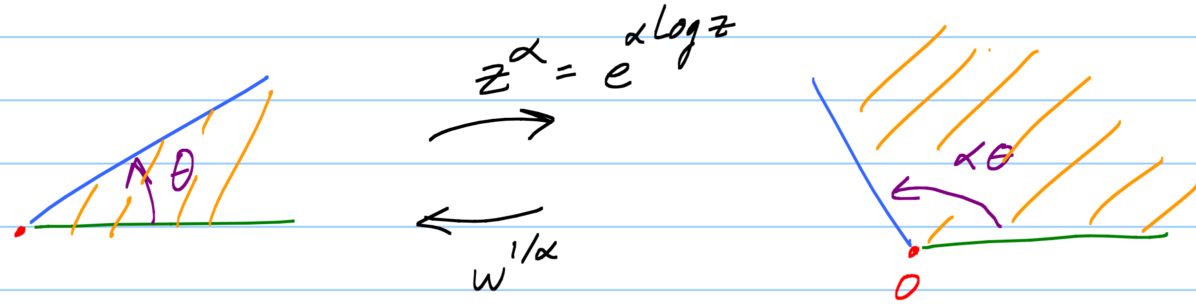
Other useful conformal maps

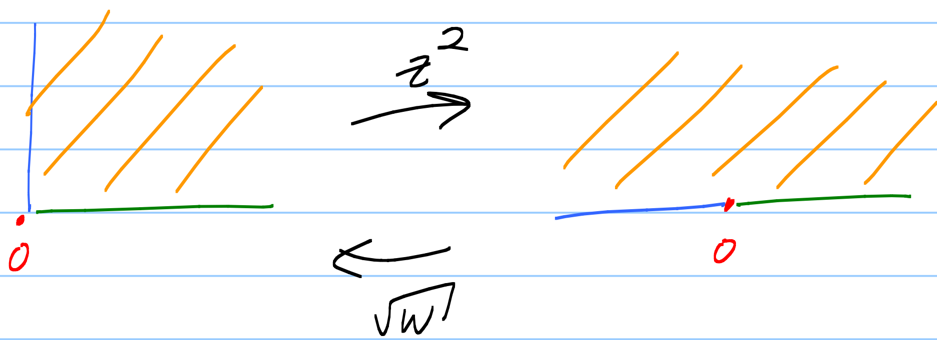


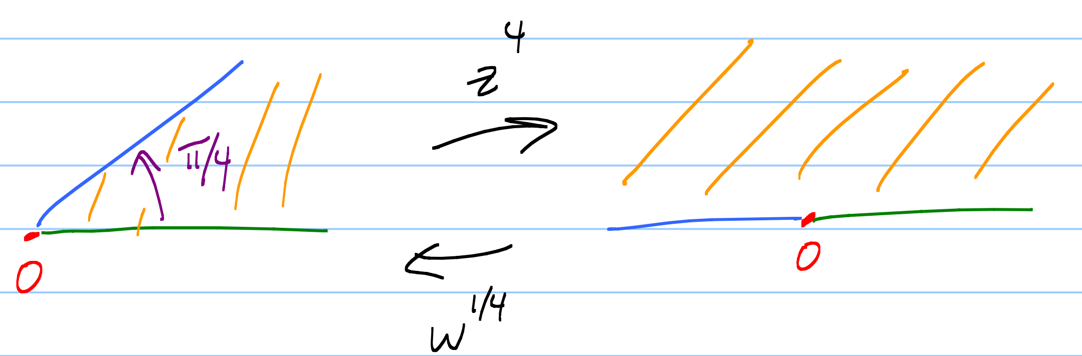


principal branch of $\sqrt{\;}$

$$\sqrt{w} = e^{\frac{1}{2}\mathrm{Log}\, w} \leftarrow \text{principal branch of log}$$







Engineers: