<u>Lecture 1</u> MA 530 Complex analysis

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MATH 750
494-1497

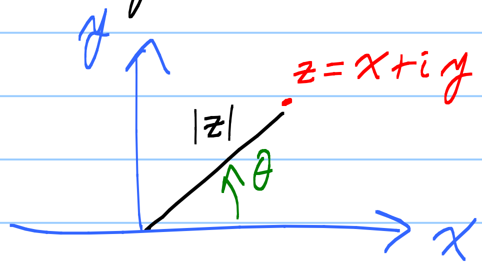
	HWK	100 pts
	Midterm Exm	100 pts
2	Midterm Exm	100 pts
	Final Exm	200 pts
		500

Quadratic formula: Complete sqr $a(x-r)^2 = c$

$c < 0$: Toss in i s.t. $i^2 = -1$.

$\mathbb{C} = \{a+bi : a, b \in \mathbb{R}\}$, Play games.

Algebra games, calculus games.



$|z|^2 = x^2 + y^2$

$\theta = \operatorname{Arg} z$, $-\pi < \theta \le \pi$

Principal argument

$\arg z = \{\operatorname{Arg} z + 2\pi n : n \in \mathbb{Z}\}$

$x = \operatorname{Re} z$

$y = \operatorname{Im} z$

Polar form: $z = re^{i\theta}$ where $e^{i\theta} = \cos\theta + i\sin\theta$

Calculus game: $e^{(a+b)} = e^a e^b$

$e^{(a+bi)} = e^a e^{bi}$

$$= e^a\left(1 + bi + \frac{(bi)^2}{2!} + \cdots\right)$$

$$= e^a\left[\left(1 - \frac{b^2}{2!} + \frac{b^4}{4!} - \cdots\right) + i\left(b - \frac{b^3}{3!} + \cdots\right)\right]$$

$$= e^a\left[\underbrace{\cos b + i\sin b}_{e^{bi}}\right]$$

Def^n: $E(x+iy) = $ "e^{x+iy}" $= e^x\cos y + ie^x\sin y$

DeMoivre's Formula: $(e^{i\theta})^N = e^{iN\theta}$

Fun thing: $1 + \cos\theta + \cos 2\theta + \cdots + \cos N\theta$

$$= \mathrm{Re}\left[1 + e^{i\theta} + e^{i2\theta} + \cdots + e^{iN\theta}\right]$$

$$= \mathrm{Re}\left[\underbrace{1 + (e^{i\theta}) + (e^{i\theta})^2 + \cdots + (e^{i\theta})^N}_{\frac{1-(e^{i\theta})^{N+1}}{1-e^{i\theta}}}\right]$$

Fourier series: Dirichlet kernel

$$\sum_{-N}^{N} e^{in\theta} = \frac{\sin(N+\frac{1}{2})\theta}{\sin\theta/2}$$

Algebra games: $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$

$(x_1, y_1)\cdot(x_2, y_2) = (x_1x_2 - y_1y_2,\ x_1y_2 + x_2y_1)$

$0 \sim (0,0)$, $1 \sim (1,0)$, $i \sim (0,1)$

$\mathbb{C}$ is a field. It is a closed field!

<u>Fund Thm Alg</u> A poly $P(z) = a_N z^N + \cdots + a_1 z + a_0$ of deg $N \geq 1$ $(a_N \neq 0)$ has a root $r \in \mathbb{C}$. $[P(r) = 0]$.

<u>Cor</u>: $P(z) = a_N \prod_{n=1}^{N} (z - r_n)$ might repeat

$= a_N \prod_{k=1}^{M} (z - r_k)^{m_k}$, $\sum_{1}^{M} m_k = N$.

Algebra game:

<u>Frobenius</u>: Every division ring gotten as a finite extension of $\mathbb{R}$ is either

$\approx \mathbb{R}$

$\approx \mathbb{C}$

$\approx \mathbb{H}$ ← quaternions

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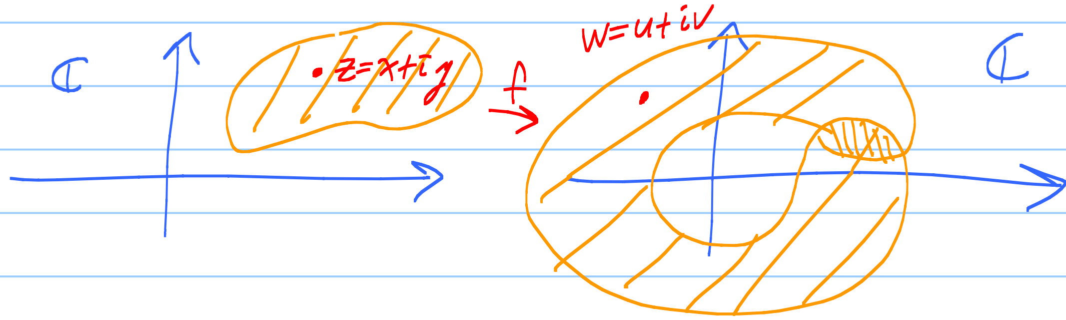


$w = f(z)$

$u + iv = f(x + iy)$ ← $u(x,y) + i v(x,y)$

$\mathbb{C} \approx \mathbb{R}^2$: $f : z \longmapsto w$ $\quad \mathbb{C} \to \mathbb{C}$

$(x,y) \longmapsto (u,v)$ $\quad \mathbb{R}^2 \to \mathbb{R}^2$

Topology on $\mathbb{C}$ $\approx$ Top on $\mathbb{R}^2$

<u>EX</u>: $w = z^2 = (x+iy)^2 = \underbrace{(x^2-y^2)}_{u(x,y)} + i\,\underbrace{2xy}_{v(x,y)}$

<u>Def'n</u>: $D_r(a) = \{z \in \mathbb{C} : |z-a|^2 < r\}$ open disc

<u>Def'n</u>: $\Omega \subset \mathbb{C}$ is <u>open</u>, if, for any $a \in \Omega$, there is an $r > 0$ s.t. $D_r(a) \subset \Omega$.

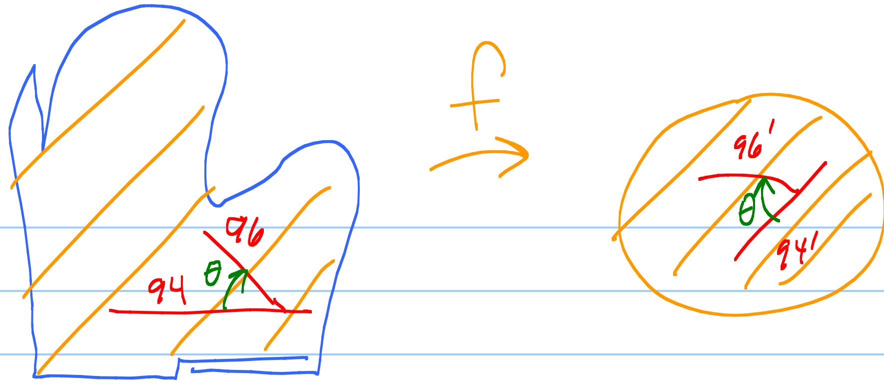
MA 530: $f : \Omega_1 \to \Omega_2$ which are <u>complex diff'ble</u>: $\lim\limits_{z\to a} \dfrac{f(z)-f(a)}{z-a}$ exists.

Call it $f'(a)$.

<u>Miracles</u>: 1) Fund Thm Alg

2) Riemann mapping thm: Ω open connected set in $\mathbb{C}$, $\Omega \neq \mathbb{C}$, Ω has no "holes".
Then $\exists$ complex diff'ble $f : \Omega \xrightarrow[\text{onto}]{1-1} D_1(0)$.
And f^{-1} is complex diff'ble too.

3) Such a map in (2) is <u>conformal</u>. It preserves angles!


96
94
θ
f
96'
94'
θ