

Math 530

Exam

1. (20 pts) Compute

$$\int_{\gamma} \frac{1}{z} dz$$

where γ is any curve in the plane that starts at $1 + i$ and ends at $2 - 2i$ and that avoids zero and the positive real axis.

Hint: Use a branch of a complex logarithm. Be sure to define it carefully.

2. (20 pts) **a)** Suppose that u is a real valued continuous function on a domain Ω . Prove that if u has no zeroes on Ω , then u is either always positive or always negative.

b) Prove that a non-vanishing real valued \mathcal{C}^2 -smooth harmonic function on the whole complex plane must be constant.

Hint: $|e^{u+iv}| = e^u$

3. (20 pts) Complete the following alternative proof of the Riemann removable singularity theorem to the one given in class. Suppose $f(z)$ is analytic and bounded by M on $D_{\epsilon}(a) - \{a\}$. Let $G(z) = (z - a)^2 f(z)$ if $z \neq a$ and define $G(a) = 0$.

a) Use a difference quotient to show that $G'(a)$ exists and is zero.

b) Show that a is a removable singularity of f (without invoking the Riemann removable singularity theorem).

4. (20 pts) Prove that if the product of two analytic functions on a domain is the zero function, then at least one of the functions must be the zero function.

Hint: The Identity theorem.

5. (20 pts) State the Schwarz lemma.