

Math 530

Exam

Each problem is worth 25 points.

1. Compute

$$\int_{\gamma} \frac{1}{z} dz$$

where γ is any curve in the plane that starts at $2 + 2i$ and ends at $-1 + i$ and that avoids the set $\{z : z = it, t \geq 0\}$ (i.e., the positive imaginary axis, including $z = 0$).

2. Suppose that u is a real valued \mathcal{C}^2 -smooth harmonic function on a domain Ω . Prove that u is either constant, or the set where the gradient of u vanishes has no limit points in Ω .

3. Compute

$$\int_0^{\infty} \frac{t}{t^7 + 1} dt$$

by integrating a complex function around a “piece of pie” with vertex angle $\frac{2\pi}{7}$ and taking limits. If you claim a limit exists, prove it.

4. Suppose that f is an entire function such that

$$\lim_{|z| \rightarrow \infty} |f(z)| = \infty.$$

- a) Explain why f can have at most finitely many zeroes.
- b) Prove that f must be a non-constant polynomial.

Hint: The in-class proof of partial fractions.