Math 530

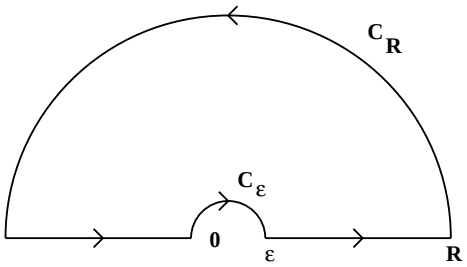
Exam 2

Each problem is worth 20 points

1. Use the contour pictured below to compute

$$\int_0^\infty \frac{\operatorname{Ln} x}{(x^2+4)^2}\, dx.$$

Justify your calculations and limits.



2. Show that if f is an analytic mapping of the unit disc into itself that has a zero of order N at the origin, then $|f(z)| \leq |z|^N$ for all z in the disc. Prove also that $|f^{(N)}(0)| \leq N!$ and determine all the functions, if any, such that $|f^{(N)}(0)| = N!$

3. Assume that f is analytic on $D_1(0) - \{0\}$ and satisfies the estimate

$$|f(z)| \leq \frac{C}{|z|^\alpha}$$

there for some constant $C > 0$ and constant α with $0 < \alpha < 1$. Prove that f has a removable singularity at $z = 0$.

4. Suppose f is analytic on $D_R(0)$ for $R > 1$ and satisfies the inequality

$$|f(z)| < |z|$$

when $|z| = 1$. Prove that f has a fixed point in the unit disc, i.e., a point a with $|a| < 1$ such that $f(a) = a$.

5. Suppose $f(z)$ is analytic above the graph of $y = x^2$, extends continuously to the parabola $y = x^2$, and maps the parabola into the real line. Explain how to the use the Schwarz reflection principle to prove that f extends analytically to an open set containing the parabola.