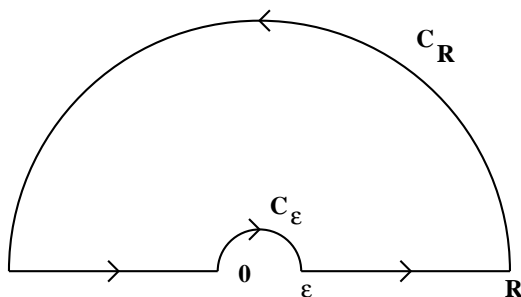


Math 530**Exam 2**

1. Assume that $a > 0$. Use the contour below to prove that

$$\int_0^\infty \frac{\operatorname{Ln} x}{(x^2 + a^2)} dx = \frac{\pi}{2a} \operatorname{Ln} a.$$

Justify your calculations and limits and define your branches.



2. Assume that $b > 1$. Use the residue theorem to show that

$$\int_0^{2\pi} \frac{d\theta}{b - \cos \theta} = \frac{2\pi}{\sqrt{b^2 - 1}}$$

3. Show that if f is an analytic mapping of the unit disc into itself such that $f(a) = 0$, then

$$|f(z)| \leq \left| \frac{z - a}{1 - \bar{a}z} \right|$$

for all z in the disc. What is the supremum of $|f'(a)|$ as f ranges over all such maps? Is the supremum attained by a map in the class? Explain.

4. Find an analytic function that maps the quarter disc $\{re^{i\theta} : 0 < r < 1, 0 < \theta < \frac{\pi}{2}\}$ one-to-one onto the vertical strip $\{z : 0 < \operatorname{Re} z < 1\}$.
(You can express your answer as a composition of maps.)

5. Use Rouché's theorem to prove that there are no monic polynomials

$$P(z) = z^n + a_{n-1}z^{n-1} + \cdots + a_1z + a_0$$

satisfying $|P(z)| < 1$ when $|z| = 1$.