Math 530
Midterm exam

1. (20 pts) Given a complex polynomial $P(z)$ of degree $N \geq 1$, there exist real constants $0 < a < A$ and $R > 0$ such that the *basic polynomial estimate*,

$$a|z|^N \leq |P(z)| \leq A|z|^N$$

holds for $|z| > R$. Show that, if an entire function $f(z)$ satisfies the right hand side of the basic polynomial estimate, it must be a polynomial of degree N or less, and if it satisfies the left hand side, it must be a polynomial of degree N or more.

2. (20 pts) Prove that the set where the gradient of a nonconstant real valued harmonic function on a domain vanishes is discrete.

3. (20 pts) Show that the open mapping theorem implies the maximum modulus principle.

4. (20 pts) Let C denote the unit circle parameterized in the counterclockwise sense. Given ϵ with $0 < \epsilon < 1$, compute

$$\int_C \frac{1}{z^2 + \epsilon z - 1}\, dz.$$

5. (20 pts) Let $f(x) = 1/(x^4 + 1)$. Compute the Fourier transform

$$\hat{f}(s) = \int_{-\infty}^{\infty} f(x) e^{isx}\, dx$$

for all real values of s. Explain your methods and back any statements you make about limits with proofs.