

Math 530

Final Exam

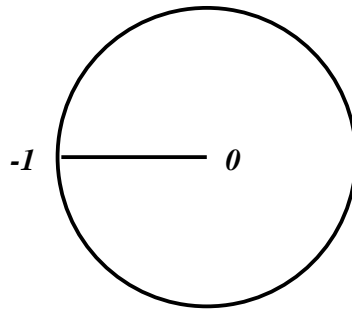
Each problem is worth 40 points

1. Calculate

$$\int_0^\infty \frac{\sqrt{x}}{(x^2 + 1)} dx.$$

Explain your reasoning and prove any limits that you use in your work.

2. Let Ω denote the slit disk $D_1(0) - (-1, 0]$. Find a one-to-one conformal mapping of Ω onto the vertical strip $\{z : 0 < \operatorname{Re} z < 1\}$. (Express your answer as a composition of maps, but don't compute the composition.)



3. Prove that a real valued function that is harmonic on the whole complex plane and that has no zeroes must be constant.
4. Prove that if f is an analytic map of the unit disk into itself with $f(0) = 0$ and $f'(0) = 0$, then $|f''(0)| \leq 2$. Furthermore, if $|f''(0)| = 2$, then $f(z) = \lambda z^2$ where λ is a complex constant of unit modulus.
5. What are the possible values of the integral

$$\int_\gamma \frac{z}{z^2 + 1} dz,$$

where γ is a path that starts at $z = -1$ and ends at $z = 2$ without passing through the points $\pm i$. Explain.