Math 530

Practice problems

1. What is the radius of convergence of the power series centered at zero for the function $1/(z-1-i)^{10}$?

2. Given a complex polynomial $P(z)$ of degree $N \geq 1$, there exist real constants $0 < a < A$ and $R > 0$ such that the *basic polynomial estimate*,

$$a|z|^N \leq |P(z)| \leq A|z|^N$$

holds for $|z| > R$. Show that, if an entire function $f(z)$ satisfies the right hand side of the basic polynomial estimate, it must be a polynomial of degree N or less. Show that if an entire function satisfies the left hand side, it must be a polynomial of degree N or more.

3. Suppose that f is an analytic function on the unit disc such that $|f(z)| < 1$ for $|z| < 1$. Prove that if f has a zero of order n at the origin, then

$$|f(z)| \leq |z|^n$$

for $|z| < 1$. How big can $|f^{(n)}(0)|$ be?

4. Show that if f is an analytic mapping of the unit disk into itself such that $f(a) = 0$, then

$$|f(z)| \leq \left|\frac{z-a}{1-\bar{a}z}\right|$$

for all z in the disk.

5. Prove that if h_1 and h_2 are two analytic functions on a domain Ω such that $h_1^N \equiv h_2^N$ for some positive integer N, then there is an N-th root of unity λ such that $h_1 = \lambda h_2$ on Ω.

6. Show that an isolated singularity of $f(z)$ cannot be a pole of $\exp f(z)$.