1

$$S_a u = \sum c_{in} S_{a_i}^n + \sum d_{jm} L_{b_j}^m$$

$$u = \sum c_{in} \frac{S_{a_i}^n}{S_a} + \sum d_{jm} \overline{\left(\frac{S_{b_j}^m}{L_a}\right)} = h + \overline{H} \leftarrow \text{rational}$$

$$\frac{\partial u}{\partial n} = i h' T - i \overline{H'} \overline{T} \leftarrow \text{rational on } b\Omega \text{ too}$$

$\mathbb{C}^n$ Poincaré $\mathbb{B}_n \not\leftrightarrow \Delta^n$

$$\mathbb{B}_n = \{(z_1, \ldots, z_n) : \sum_{j=1}^n |z_j|^2 < 1\}$$

$\text{Aut}(\mathbb{B}_n)$ is not hard to determine.
It is transitive, meaning, given $a, b \in \mathbb{B}_n$, $\exists\, \varphi \in \text{Aut}(\mathbb{B}_n)$ such that $\varphi(a) = b$.

Get Bergman kernel for $\mathbb{B}_n$ using automorphisms
Use $K(z, 0) = \frac{1}{\text{Vol}(\mathbb{B}_n)}$. $\quad K(z,w) = \frac{c}{(1 - z \cdot \bar{w})^{n+1}}$

For polydisc $\Delta^n = \Delta \times \Delta \times \cdots \times \Delta$,

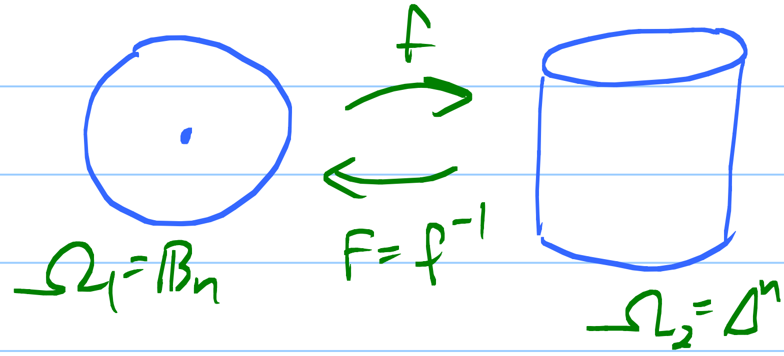
$$\varphi(z_1, \ldots, z_n) = \left(e^{i\theta_1} \frac{z - a_1}{1 - \bar{a}_1 z}, \ldots, e^{i\theta_n} \frac{z - a_n}{1 - \bar{a}_n z}\right)$$

are the automorphisms of Δ^n. Get

$$K(z,w) = \frac{1}{\pi^n} \prod_{j=1}^n \frac{1}{(1 - z_j \bar{w}_j)^2}.$$

Thm: $\mathbb{B}_n$ and Δ^n are biholo inequiv, $n>1$.

Pf: Suppose $f : \mathbb{B}_n \to \Delta^n$ biholo



$u = \det\left[\frac{\partial f_i}{\partial z_j}\right]$

$U = \det\left[\frac{\partial F_i}{\partial w_j}\right]$

$$(*)\quad \boxed{u(z)\, K_2(f(z), w) = K_1(z, F(w))\, \overline{U(w)}}$$

Step 1: May assume $f(0)=0$. (Use trans auto grps)

Step 2: Let $w=0$.

$$u(z) \cdot \frac{1}{\mathrm{Vol}(\Delta^n)} = \frac{1}{\mathrm{Vol}(\mathbb{B}_n)} \cdot \overline{U(0)}$$

See $u(z) \equiv \text{const}$.

Step 3: Diff $(*)$ wrt $\bar{w}_j$

$$u(z)\, K_2^{\bar{j}}(f(z), w) = K_1(z, F(w))\, \overline{U^{(j)}(w)} + \sum K_1^{(\bar{k})}(z, F(w))\, \overline{U(w)}\, \overline{\frac{\partial F_k}{\partial w_j}}$$

Plug in $w=0$:

$$u(z)\, f_j(z) = (\text{const}) + (\text{linear})$$

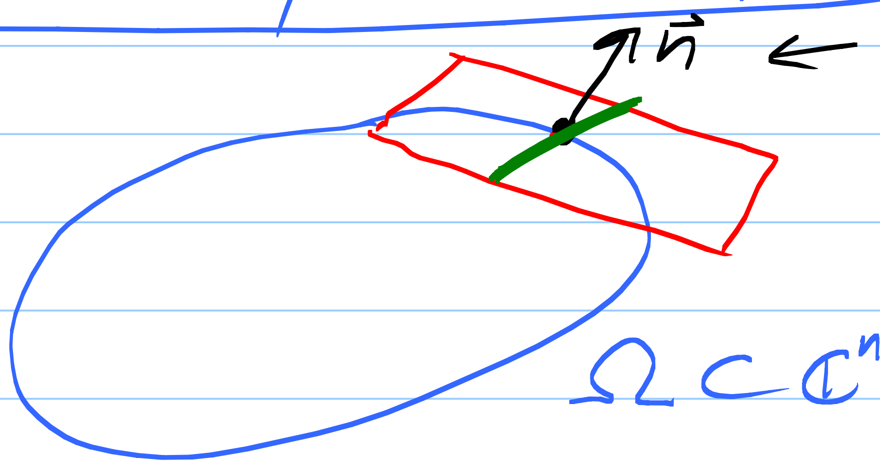
See $f_j(z)$ is linear! No way!

$\mathbb{B}_n$ smooth bndry. Δ^n not.

No Riemann Mapping Thm in $\mathbb{C}^n$

Riemann Mapping almost Thm: Suppose Ω is a bounded C^2 strictly pseudoconvex in $\mathbb{C}^n$ with transitive automorphism group. Then $\Omega \cong \mathbb{B}_n$.

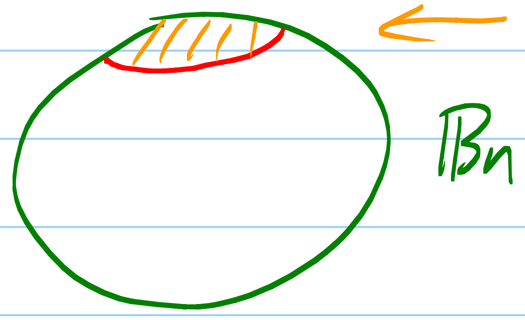
Tangential Cauchy Riemann Eqns:



← multiply $\vec{n}$ by $\mathbb{C}$. Get 1-dim^e complex line.

Orthogonal complement of $\mathbb{C}\cdot\vec{n}$ in $\mathbb{C}^n$ is a complex $n-1$ dimensional subspace of the tangent space in $\mathbb{R}^{2n}$. Take a complex direction in this subspace and push it a little inside Ω. Given holo h that extends C^1 smooth to $b\Omega$, see h satisfies C-R eqns in that direction <u>on the boundary</u>.

<u>Fact</u>:



h extends holo to a solid polar chunk of $\mathbb{B}_n$.

<u>Problem</u>: Given $f: b\Omega_1 \xrightarrow{\text{cont.}} b\Omega_2$ locally and satisfying tangential Cauchy-Riemann eqns, show that if $b\Omega_1$ and $b\Omega_2$ are C^∞ smooth, then so is f.

Consequence of C^∞ smoothness: Diff. geometric invariants would get preserved. See "most" domains are biholo inequiv.